Edited Expectations

Washington State Board of Education
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Strategic Teaching
Better learning by design
1573 Millersville Road
Millersville, MD 21108

We are pleased to submit suggested revisions to the Washington State Board of Education for the grade K–8 mathematics standards. We believe these new standards compare favorably with the best in the world. The standards are now what leaders in the state have insisted they must be – clear, rigorous and measurable.

More importantly, we believe they are organized and written in a way that will provide clarity to Washington teachers, students and their parents. In the end, standards only matter if they result in students meeting these expectations.

Our suggestions build on a strong foundation. The Office of the Superintendent of Public Instruction (OSPI) crafted a good set of standards that balanced the conceptual strength in its previous document with clear expectations for computational fluency. Washington is the first state to make a serious attempt to organize by topics that focus student learning, rather than mathematics strands. We work on standards around the nation, and OSPI's effort should be seen as a model for other states.

We were very much aware of how much work had been done in the past several months by a wide range of Washington educators and parents. We tried hard to honor that work, and limit revisions to only those we saw as necessary. As you will see below, we changed the document's structure only in one content area – geometry – and that was mostly to reduce a content overload in third grade.

We had two main goals:

First, to improve the clarity of language throughout the standards.

Second, to remove instructional specificity – pedagogy -- from the standards.

We think we have accomplished both.

In many cases, you will see that we made no changes at all to the standards. When we did rewrite, in some cases in was to make the standard more understandable or the expectation more precise, as in the following situation.

- 1.4.F Name standard units of time: day, week, month.
- 1.4.F (Revised) Name the days of the week and the months of the year and use a calendar to determine a day or month.

And sometimes we rewrote to focus on *what* not *how*. In other words, we wrote to focus on the mathematical content not the instruction, although we generally recommend that this information be retained in *Comments.*

- 5.2.B Represent addition and subtraction of decimals using words, numbers, pictures, and physical materials, and translate among representations.
- 5.2.B Represent addition and subtraction of decimals using place value models and connect the representation to the related number sentence.

In a few, selected instances, we choose to make global changes. For example, in the original set of standards, students often were asked to "justify the solution" in a "contextual situation." Our change for clarity was to have students "verify the solution" in "word problems." We ask that OSPI remove "as needed" and "when appropriate" from the document. These phrases that add little clarity or specificity to the document.

We also eliminated the modifying phrase "at least" from the standards. We believe that standards by intention define what is expected of all students, and students should do *at least* those things. We expect teachers should make every effort to extend their expectations of students. Kindergarten students who can count up to 120 (the standard) should be taught to count to 200; grade 7 students who can solve one-step equations (the standard) are ready for simple two-step equations.

There are some topics we have not yet clarified to our satisfaction including calculator use and the interplay of *Core Processes and Additional Key Topics.* We expect to do so in revisions to the introductory material, which we will submit with Algebra II edits in June.

As we have said from the beginning: no set of standards is perfect. While we think this one is very close, we predict that there are those who will take exception.

Some will say our revisions make the standards too rigorous. We do not believe this is the case. We considered the research regarding developmentally appropriate expectations and we looked hard at what happens in other places.

Some will say our revisions create standards that are not rigorous enough. Again, we do not believe this is the case. We grant there are instances of topics that exist in Indiana or California, for example, a lower grade level, but never more than one grade level and never without reason. For instance, we now place perimeter in grade four rather than three. We do this consciously to ensure there is sufficient time in grade three for students to master the fundamentals, particularly to master addition and subtraction and to do significant work with multiplication, division and fractions.

We would counter the "not rigorous enough" argument by pointing out we moved other topics, such as two-dimensional representations of three-dimensional objects, to earlier grade levels. Again, we thoughtfully placed standards with consideration for the full content load of the grade level and with attention to the connections to other mathematical content that is taught during the year.

Our purpose is to ensure that students master the standards and thoughtfully assimilate the content, rather than race through without absorbing or retaining information.

Next week the abbreviated benchmarking report will be released. Using the same methodology as in the original standards review, we are comparing the March 5 version of Washington's standards to California, Indiana, Curricular Focal Points, Singapore, and the *Final Report* of the National Math Panel at grades 2, 4, and 8. We are also benchmarking the most appropriate aspect of high school to California, Indiana, Singapore, and the Washington College Readiness Standards. This report will shed much more light on the area of topical alignment.

We considered carefully the work of the National Mathematics Advisory Panel's *Final Report* as we reviewed OSPI's standards. Washington expects its students to master content at very similar grade levels, as is shown by the chart on the following page.

There are differences, but we cared more about respecting the existing good standards and about ensuring that Washington have a set of standards able to support the teaching of all important mathematics than we cared about having a set of standards that matched exactly to the National Math Panel. Algebra, while critical and the well-recognized gate keeper, is not all of mathematics. The National Mathematics Advisory Panel was tasked with looking at just that aspect, not at a total mathematics program. It is completely candid in saying that its recommendations are focused on Algebra. Our intended goal, is a document that ensures Washington students do indeed master the mathematics they need to succeed in school and beyond.

Content area	National Math Panel Grade Level	Washington Expectation Grade Level
Automatically recall addition and related subtraction facts.	3	3
Automatically recall multiplication and related division facts.	4	4
Identify and represent fractions and decimals.	4	4
Compare fractions and decimals on a number line or with other common representations of fractions and decimals.	4	4
Understand why and how (finite) decimal numbers are fractions.	5	5
Be proficient with multiplication and division of whole numbers.	5	5
Be proficient with comparing fractions and decimals and common percents	5	6
Solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides	5	5
Add and subtract fractions and decimals.	5	5
Estimate the size of fractions, decimals and percents.	5	5
Multiply and divide fractions and decimals.	6	6
Compute with all operations using positive and negative integers.	6	7
Solve problems involving perimeter and area.	6	4
Solve problems involving surface area and volume	6	6
Use formulas to determine perimeter, area, volume, and surface area.	6	6
Find unknown lengths, angles, and areas	6	6

Compute with all operations using positive and negative fractions	7	7
Solve problems involving percent.	7	7
Solve problems involving ratio	7	6
Solve problems involving rate.	7	6
Solve problems involve proportion.	7	7
Be familiar with the relationship between similar triangles.	7	7
Be familiar with the concept of the slope of a line.	7	7
A solid understanding of the commutative, distributive, and associative properties	8	8
Develop proportional reasoning.	8	7
Understand exponential forms of numbers.	8	8

For those students able to move faster and interesting in taking Algebra in grade 8, OSPI has committed to producing a set of standards, this summer, that compress grades 6 to 8 into a two year program. Given the current K–6 and 7–8 structure of many school systems, we also urge OSPI to consider an additional version that compresses grades 7–8 into one year.

The revised *Expectations* are organized with the original *Expectation* in the left column. If there is no change, that is noted. If any change is suggested, the replacement *Expectation* is put in the right hand column. If the Expectation has been moved to another place in the standards, it is crossed out and its new location noted. Additionally, in the cell with the new *Expectation,* we note those places where we believe that changes should be made in the *Examples and Comments,* along with any pertinent details.

Kindergarten

K.1. Core Content: *Whole numbers (Numbers, Operations)*

Students begin to develop basic notions of numbers and use numbers to think about objects and the world around them. They practice counting objects in sets, and they think about how numbers are ordered by showing the numbers on the number line. By putting together and taking a part simple numbers, students lay the groundwork for learning how to add and subtract. Understanding numbers is perhaps the most central idea in all of mathematics, and if students build and maintain a strong foundation of number sense and number skills, they will be able to succeed with increasingly sophisticated numerical knowledge and skills from year to year.

Current	Edited *Expectations* and *Comments*
K.1.A Rote count by ones forward from 1 to at least 100 and backward from any number in the range of 10 to 1.	**K.1.A Rote count by ones forward from 1 to 100 and backward from any number in the range of 10 to 1.**
K.1.B Read aloud numerals from 0 to at least 31.	**K.1.B Read aloud numerals from 0 to 31.**
K.1.C Fluently compose and decompose numbers to at least 5.	Language similar to this **should** be added to the *Comments*: – Students should recognize the number of missing objects without counting. **Consider** adding language like: – The words compose and decompose come form the work of Carpenter et.al., who described that young students acquire knowledge of small numbers by breaking them into parts. This understanding (sometimes called part/part/whole) is not only a pedagogical issue: it is the bridge between counting on and knowing number combinations. It is how instant recognition of small numbers develops and so leads naturally to understanding fact families.
K.1.D Order numerals from 1 to at least 10.	**K.1.D Order numerals from 1 to 10.**
K.1.E Count objects in a set of up to 20, and count out a specific number of up to 20 objects from a larger set.	No change
K.1.F Compare two sets of up to 10	**K.1.F Compare two sets of up to 10 objects each and say whether the number of**

objects each and explain why the number of objects in one set is equal to, greater than, or less than the number of objects in the other set.	**objects in one set is equal to, greater than, or less than the number of objects in the other set.**
K.1.G Locate numbers from 1 to at least 31 on the number line.	No change
K.1.H Describe numbers from 1 to 9 using 5 as a benchmark number.	**K.1.H Describe a number from 1 to 9 using 5 as a benchmark number.**

K.2. Core Content: *Patterns and operations (Operations, Algebra)*
Students learn what it means to add and subtract by joining and separating sets of objects. Working with patterns helps them strengthen this understanding of addition and subtraction and moves them toward the important development of algebraic thinking. Students study simple repetitive patterns in preparation for increasingly sophisticated patterns that can be represented with algebraic expressions in later grades.

Existing	Edited *Expectations* and *Comments*
K.2.A Recognize, extend, and create simple repetitive patterns, and identify a missing element in a pattern.	**K.2.A Copy, extend, describe, and create simple repetitive patterns~~.~~** These words **should** be added to the *Comments*. – Students can complete these activities with specified patterns of the type AB, AAB, AABB, ABC, etc. The language in the examples **should be edited**, leaving the graphics unchanged: – Make a type AB pattern of squares and circles with one square, one circle, one square, one circle, etc. – Here is a type AAB pattern using colored cubes: red, red, blue, red, red, blue, red, red. What comes next? – A shape is missing in the type AB pattern below. What is it?
K.2.B Translate a pattern among sounds, symbols, movements, and physical objects.	No change

K.2.C Model addition by joining sets of objects that have 10 or fewer total objects when joined; model subtraction by separating a set of 10 or fewer objects.	No change
K.2.D Describe a situation that involves the actions of joining (addition) or separating (subtraction) using words, pictures, objects, or numbers.	No change

K.3. Core Content: *Objects and their locations (Geometry/Measurement)*
Students develop basic ideas related to geometry as they name simple two- and three-dimensional figures and find these shapes around them. They expand their understanding of space and location by describing where people and objects are. Students sort and match shapes as they begin to develop classification skills that serve them well in both mathematics and reading—matching numbers to sets, shapes to names, patterns to rules, letters to sounds, and so on.

Current	Edited *Expectations* and *Comments*
K.3.A Identify, name, and describe circles, triangles, rectangles, squares (as special rectangles), cubes, and spheres, regardless of size or orientation.	**K.3.A Identify, name, and describe circles, triangles, rectangles, squares (as special rectangles), cubes, and spheres.** *Comments* similar to the following **should** be added: – Students should be encouraged to talk about the characteristics (i.e., round, having four corners) of the various shapes and to identify them in a variety of contexts despite their location, size, and orientation. Having students identify these shapes on the playground, in books, and on clothing develops their ability to generalize the characteristics of each shape.
K.3.B Identify circles, triangles, rectangles, squares, cubes, and spheres in a variety of contexts.	~~K.3.B Identify circles, triangles, rectangles, squares, cubes, and spheres in a variety of contexts.~~ This expectation is now part of the *Comments* for K.3.A
K.3.C Sort shapes using a sorting rule	**Consider** modifying the *Comments* to include more attributes related to shape:

and explain the sorting rule.	– Sort shapes using a sorting rule such as the name of shape, size, and number of sides, and explain the sorting rule.
K.3.D Describe the location of one object relative to another object using words such as *in*, *out*, *over*, *under*, *above*, *below*, *between*, *next to*, *behind*, and *in front of*.	No change

K.4. Additional Key Content *(Geometry/Measurement)*
Students informally develop early measurement concepts. This is an important precursor to Core Content on measurement in later grades, when students measure objects with tools. Solving measurement problems connects directly to the student's world and is a basic component of learning mathematics.

Current	Edited *Expectations* and *Comments*
K.4.A Identify measurable attributes, such as length, weight, and capacity, and use these attributes to make direct comparisons.	**K.4.A. Make direct comparisons using measurable attributes such as length, weight, and capacity.** **Consider** *Comment* replacements like: – Students should be encouraged to use language such as longer than, shorter than, taller, heavier than, lighter than, and holds more or holds less than.

K.5. Core Processes: *Reasoning, problem solving, and communication*
Students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Problems at this level emphasize counting and activities that lead to emerging ideas about addition and subtraction. Students begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?"

Current	Edited *Expectations* and *Comments*
K.5.A Identify the question(s) asked in a problem and any other questions that need to be answered in order to solve the problem.	**K.5.A Identify the question(s) asked in a problem.**

K.5.B Identify the given information that can be used to solve a problem.	No change
K.5.C Recognize when additional information is required to solve a problem.	No change
K.5.D Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.	No change
K.5.E Identify the answer(s) to the question(s) in a problem.	**K.5.F Solve the problem.**
K.5.F Describe how a problem was solved.	No change
K.5.G Determine whether a solution to a problem is reasonable.	No change

Grade 1

1.1. Core Content: *Whole number relationships (Numbers, Operations)*
Students continue to work with whole numbers to quantify objects. They consider how numbers relate to one another. As they expand the set of numbers they work with, students start to develop critical concepts of ones and tens that introduce them to place value in our base ten number system. An understanding of how ones and tens relate to each other allows students to begin adding and subtracting two-digit numbers, where thinking of ten ones as one ten and vice versa is routine**.**

Language similar as the following should be added to this paragraph: "Some students will be ready to work with numbers larger than those identified in the *Expectations* and should be given every opportunity to do so."

Current	Edited *Expectations* and *Comments*
1.1.A Count by ones forward and backward from 1 to at least 120, starting at any number, and count by twos, fives,	**1.1.A Count by ones forward and backward from 1 to 120, starting at any number, and count by twos, fives, and tens to 100.**

and tens to at least 100.	In the *Comments* it **should** read: – Once a student counts to 120 consistently..."
1.1.B Name the number before or after any number given verbally up to at least 120.	**1.1.B Name the number one less or one more than any number given verbally up to 120.**
1.1.C Read aloud numerals from 0 to 1,000.	No change
1.1.D Order objects or events using ordinal numbers.	*Comments* such as the following **should** be added: – Students use ordinal numbers to twentieth ...
1.1.E Write, compare, and order numbers to at least 120 using the words *equal to*, *greater than*, *less than*, *greatest*, and *least* when appropriate.	**1.1.E Write, compare, and order numbers to 120.** **Consider** adding language to the *Comments* such as: – Students put numbers in lists or talk about the relationships using the words *equal to*, *greater than*, *less than*, *greatest*, and *least*.
1.1.F Fluently compose and decompose numbers to at least 10.	1.1.F Fluently compose and decompose numbers to 10. Language **similar** to the following **should** be added to the explanation *Comments*: – Students put together and take apart whole numbers as a precursor to addition and subtraction. – Add the parenthetical comment, as shown, to this example: "Here are twelve coins. I will hide some. If you..." with one or two examples of math facts **(i.e., this is an example of going beyond expectation.)**
1.1.G Group numbers into tens and ones in more than one way and demonstrate that the total remains the same.	**1.1.G Group numbers into tens and ones in more than one way.** **Consider** adding language to the comments such as: – Students demonstrate that the value of a number remains the same regardless of how it is grouped.
1.1.H Group and count objects by tens, fives, and twos.	No change

1.1.I Use words, objects, or pictures to demonstrate whether a given whole number is odd or even.	**1.1.I Classify a number as odd or even and demonstrate it is odd or even.**
	
	The following change in the *Comments* **should** be made:
	– Students use words, objects, or pictures to demonstrate that a given number is odd or even.
	  – 13 is odd because you can't make two equal piles of counters.
	  – 20 is even because every counter matches with another counter.

1.2. Core Content: *Addition and subtraction (Operations, Algebra)*
Students learn how to add and subtract, when to add and subtract, and how addition and subtraction relate to each other. Understanding that addition and subtraction undo each other is an important part of learning these operations efficiently and accurately. Students notice patterns involving addition and subtraction, and they work with other types of patterns as they learn to make generalizations about what they observe.

Current	Edited *Expectations* and *Comments*
1.2.A Represent addition and subtraction with physical objects and symbols, and connect the representations.	**1.2.A Connect physical and pictorial representations to addition and subtraction equations.** *Comments* **similar** to this **should** be added: – The number sentence 2 + 6 = 8, could be represented by drawing a set of 2 cats + 6 cats to get a set of 8 cats. – The number sentence 9 – 5 = 4 could be represented by taking 5 objects away from a set of 9 objects.
1.2.B Describe equivalent names for the same number using the word *equals* and the equal sign (=).	**1.2.B Use the equals sign (=) and the word "equals" to indicate that two expressions are equivalent.**
1.2.C Represent addition and subtraction using movement on the number line.	**1.2.C Represent addition and subtraction on the number line.** The follow changes **should** be made in the *Comments*: – A pictorial representation of addition or subtraction on the number line. Including an example of each would be preferred.
1.2.D. Demonstrate the inverse relationship between addition and subtraction by undoing an addition	The follow changes **should** be made in the *Comments*: – An example of a word problem that requires students to show understanding of the inverse relationship is needed.

problem with subtraction and vice versa.	
1.2.E Add three or more one-digit numbers using the commutative and associative properties of addition and justify the solution	**1.2.E Add three or more one-digit numbers using the commutative and associative properties of addition.**
1.2.G Quickly recall addition facts and related subtraction facts for sums equal to at least 10.	No change
1.2.F Explain and use strategies for remembering basic addition facts and related subtraction facts for sums equal to at least 10.	**1.2.F Apply and explain strategies to compute addition facts to 18 and their related subtraction facts.** *Comments* **similar** to the following **should** be included: – Strategies for addition include, but go beyond, counting on.
1.2.H Solve and create story problems that match addition or subtraction expressions or equations using physical objects,	**1.2.H Solve and create addition and subtraction word problems.** *Comments* **similar** to the following **should** be included: – Students solve problems using objects, pictures and/or numbers.

1.2.I Recognize, extend, and create number patterns.	Language in the *Comments* **must** be changed as follows:
	
	– "Extend the pattern below and tell how you decided what numbers could come next." To "Extend the simple addition pattern below and tell how you decided what numbers come next."
	– Examples **similar** to the following **should** be included:
	  – 2, 4, 6, 8, 10...
	  – 5, 10, 15, 20, 25...

1.3. Core Content: *Geometric attributes (Geometry/Measurement)*
Students expand their knowledge of two- and three-dimensional geometric figures by sorting, comparing, and contrasting them according to their characteristics. They learn important mathematical vocabulary used to name the figures. Students work with composite shapes made out of basic two-dimensional figures as they continue to develop their spatial sense of shapes, objects, and the world around them.

Current	Edited *Expectations* and *Comments*
1.3.A Compare and sort a variety of two- and three-dimensional figures according to their geometric attributes.	**No change**
1.3.B Identify and name two- and three-dimensional figures, including those in real world contexts, regardless of size or orientation.	**1.3.B Identify and name two-dimensional figures, including those in real world contexts, regardless of size or orientation.** The *Comments* **should** be modified to be **similar** to the following and eliminate 3-dimensional references: **Figures should include circles, triangles, rectangles, squares (as special rectangles), rhombi, hexagons, and trapezoids.** **Contextual examples could include classroom clocks, flags, tissue boxes, birthday party hats, cans of vegetables, etc. Triangles should appear in many positions and orientations and should not all be equilateral or isosceles.**
1.3.C Compose and decompose common two-dimensional figures.	**1.3.C. Combine known shapes to create shapes, divide known shapes into other shapes.**

1.4. Core Content: *Concepts of measurement (Geometry/Measurement)*
Students begin to understand what it means to measure something, and they develop their measuring skills using everyday objects. They learn the difference between measuring length, weight, and capacity and use objects like toothpicks and craft sticks to measure length, marbles and a balance to measure weight, and cups filled with water or sand to measure capacity. As they practice using measurement tools to measure objects, they reinforce their numerical skills and continue to develop their sense of space and shapes.

Comments <u>similar</u> to the following <u>can be considered</u> for addition to the opening paragraph: Students learn about measurement through length. This attribute serves as the vehicle through which students come to understand that units of measure must be the equal in size, that standard-sized units exist and the approximate size of those units, that it takes more, smaller units to measure an item, and how the transitive property works. Over time they apply these same concepts of measure to other attributes such as weight and capacity.

Current	Edited *Expectations* and *Comments*
1.4.A Recognize that objects used to measure an attribute (length, weight, capacity) must have that attribute and must be consistent in size.	**1.4.A Recognize that objects used to measure an attribute (length, weight, capacity) must be consistent in size.**
1.4.B Use a variety of non-standard units to measure length.	No change
1.4.C Apply the transitive property when comparing lengths.	**1.4.C Compare lengths using the transitive property.**
1.4.D Use non-standard units to compare objects according to their capacities or weights.	No change
1.4.E Describe the relationship between the size of the measurement unit and the number of units needed to measure something.	**1.4.E Describe the connection between the size of the measurement unit and the number of units needed to measure something.**

1.4.F Name standard units of time: day, week, month.	**1.4.R Name the days of the week and the months of the year and use a calendar to determine a day or month.**
	
	The *Comments* **should** be modified with additional examples **similar** to the following:
	– In addition to reciting the order of the days and months, students answer questions such as:
	  – How many days until your birthday?
	  – What month comes next?
	  – What day was it yesterday?

1.5. Additional Key Content *(Data/Statistics/Probability)*
Students are introduced to basic ideas of statistics by collecting and visually representing data. These ideas reinforce their understanding of the Core Content areas related to whole numbers and addition and subtraction as students ask and answer questions about the data. As they move through the grades, students will continue to apply what they learn about data, making mathematics relevant and connecting numbers to applied situations.

Current	Edited *Expectations* and *Comments*
1.5.A Represent data using tallies, tables, picture graphs, and bar-type graphs.	No change

1.5.B Analyze information by asking and answering questions about data.	**1.5.B Ask and answer comparison questions about data**.
	
	The *Comments* **should** be modified with additional examples **similar** to the following:
	– Students answer questions like:
	  – Which has the most?
	  – Which the least.
	  – How many more?

1.6. Core Processes: *Reasoning, problem solving, and communication*
Students further develop the concept that doing mathematics involves solving problems and discussing what they did to solve them. Problems in first grade emphasize addition, subtraction, and solidifying number concepts, and sometimes include precursors to multiplication. Students continue to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Why did you do that?" and "How do you know that?" Students begin to build their mathematical vocabulary as they use correct mathematical language appropriate to first grade.

Current	Edited *Expectations* and *Comments*
1.6.A Identify the question(s) asked in a problem and any other questions that need to be answered in order to solve the problem.	**1.6.A Identify the question(s) asked in a problem.**
1.6.B Identify the given information that can be used to solve a problem.	No change
1.6.C Recognize when additional information is required to solve a problem	No change
1.6.D Select from a variety of problem-solving strategies and use one or more strategies to solve a problem.	**1.6.D Solve the problem**

1.6.E Identify the answer(s) to the question(s) in a problem.	No change
1.6.F Describe how a problem was solved.	No change
1.6.G Determine whether a solution to a problem is reasonable.	No change

Grade 2

2.1. Core Content: *Place value and the base ten system (Numbers)*

Students refine their understanding of the base ten number system and use place value concepts of ones, tens, and hundreds to understand number relationships. They become fluent in writing and renaming numbers in a variety of ways. This fluency, combined with the understanding of place value, is a strong foundation for learning how to add and subtract two-digit numbers.

Current	Edited *Expectations* and *Comments*
2.1.A Count by tens or hundreds forward and backward from 1 to 1,000, starting at any number.	No change
2.1.B Represent numbers to at least 1,000 in different ways using written words, numerals, and models, and translate among representations.	**2.1. Connect place value models with their numerical equivalents to 1000.** The following **should** be added to the *Comments:* – Pictorial examples that use grid paper and base 10 blocks
2.1.C Identify the ones, tens, and hundreds places in a number and the digits occupying them.	No change
2.1.D Compose and decompose two- and three-digit numbers based on the values represented by the digits used to write the number.	**2.1.D Write three-digit numbers in expanded form.**

2.1.E Describe a number up to 1,000 in terms of how many ones, how many tens, or how many hundreds it includes.	**2.1.E Group three-digit numbers into hundreds, tens and ones in more than one way.** **Consider** adding the following or **similar** examples to the *Comments*: 3 hundreds + 19 tens + 3 ones = 4 hundreds + 8 tens + 13 ones
2.1.F Compare and order numbers from 0 to at least 1,000 using the words *equal to*, *greater than*, *less than*, *greatest*, or *least* when appropriate.	**2.1.F Compare and order numbers from 0 to 1,000.** The following language **should** be added to the *Comments*: – Students use the words *equal to*, *greater than*, *less than*, *greatest*, or *least and their matching symbols, (=), (<), and (>).*

2.2. Core Content: *Addition and subtraction (Operations, Geometry/Measurement, Algebra)*
Students focus on what it means to add and subtract as they become fluent with single-digit addition and subtraction facts and develop addition and subtraction procedures for two-digit numbers. Students make sense of these procedures by building on what they know about place value, number relationships, and putting together or taking apart sets of objects. This is students' first time to deal formally with step-by-step procedures (algorithms)—an important component of mathematics that allows students to use a generalizable technique in similar situations. Students begin to use estimation to determine if their answers are reasonable.

Current	Edited *Expectation* and *Comment*
2.2.A Explain and use strategies for remembering addition and subtraction facts to 20.	~~2.2.A Explain and use strategies for remembering addition and subtraction facts to 20.~~ This is included in the revised 1.2.F.
2.2.B Quickly recall basic addition facts and related subtraction facts for sums through 20.	No change
2.2.C Solve a variety of addition and subtraction problems and justify the solutions.	**2.2.C Solve addition and subtraction word problems that involve joining, separating and comparing and verify the solution.** The following **language should be deleted** from the *Comments*: – "Students should be able to use all of these representations as needed."
2.2.D Add and subtract two-digit	**2.2.D Add and subtract two-digit numbers efficiently and accurately using a procedure**

numbers efficiently and accurately using a procedure that can be generalized (an algorithm) and explain why the procedure works.	**that works with all 2-digits numbers and explain why the procedure works.**
2.2.E Add and subtract two-digit numbers mentally and explain the strategies used.	No change
2.2.F Determine whether a sum or difference is reasonable.	**2.2.F Estimate sums and differences.**
2.2.G Extend and create patterns that can be generated by addition, including growing patterns, and explain the rules used to generate the patterns.	**2.2.G Extend, create, and state the rule for patterns that can be generated by addition.** The first example **should be removed** from *Comments*, because it no longer applies to the revised *Expectation*: ~~2 + 3 = 5; 20 + 30 = 50; 200 + 300 = 500~~
2.2.H Solve equations in which the unknown number and the equal sign appear in a variety of positions.	**2.2.H Solve equations in which the unknown number appears in a variety of positions.**
2.2.I Name the standard United States coins and their values and equivalents and write their values using the $ sign and the ¢ sign.	**2.2.I Name each standard U.S. coin, write its value using the $ sign and the (¢) sign, and name combinations of other coins with the same total value.**
2.2.J Determine the value of a collection of coins totaling less than $1.00.	No change

2.3. Core Content: *Measurement (Geometry/Measurement)*
Students understand the process of measuring length and progress from measuring length with objects such as toothpicks or craft sticks to the more practical skill of measuring length with standard units and tools such as rulers, tape measures, or meter sticks. Students, well acquainted with two-digit numbers by this point, tell time on different types of clocks.

Current	Edited *Expectation* and *Comment*
2.3.A Identify objects that represent standard units and use them to measure length.	**2.3.A: Identify objects that represent <u>or approximate</u> standard units and use them to measure length.** **Consider** inserting language in the **Comments** similar to: – Students find and use approximations for standard length units, like a paper clip whose length is about an inch, or the width of a particular student's thumbnail that might be about a centimeter. They might also use commonly available classroom objects like inch tiles or centimeter cubes.
2.3.B Estimate length using metric and customary units.	**2.3.B Estimate length using metric and U.S. customary units.**
2.3.C Use metric and customary units and tools to measure length.	**2.3.C Measure length to the nearest whole unit in both metric and U.S. customary units.**
2.3.D Describe the relationship among standard units of time: minutes, hours, days weeks, months, and years.	**2.3.D Describe the relative size among minutes, hours, days, weeks, months, and years.**
2.3.E Use both analog and digital clocks to tell time to the minute.	No change

2.4. Additional Key Content *(Numbers, Operations, Geometry/Measurement, Data/Statistics/Probability)*
Students make predictions and answer questions about data as they apply their growing understanding of Core Content related to numbers and the operations of addition and subtraction. They extend their spatial understanding of Core Content in geometry developed in kindergarten and grade one by solving problems involving two- and three-dimensional geometric figures. Students are introduced to a few critical concepts that will become Core Content in grade three. Specifically, they learn what a fraction is and begin to work with multiplication and division.

Current	Edited *Expectation* and *Comment*
2.4.A Use the attributes of geometric figures to solve spatial problems.	~~2.4.A Use the attributes of geometric figures to solve spatial problems.~~ **2.4.A Solve problems involving properties of two- and three-dimensional figures.** The following language **should** be includes in the *Comments:* – A critical component in the development of students' spatial and geometric understanding is to be able to solve problems involving the properties of figures. At the primary level, students must move from judging plane and space shapes by their appearance as whole shapes to focusing on the relationship of the sides, angles or faces. At the same time, students must learn the language important in describing shapes according to their essential characteristics. Later, they will describe properties of shapes in a logical order, which prepares them for the more formal and abstract study of geometry typical of future mathematics study. Examples: – How many different ways can you fill the outline of the figure with pattern blocks? What is the greatest number of blocks you can use? The least number? Can you fill the outline with every whole number of blocks between the least number of blocks and the greatest number of blocks? – Build a figure or design out of five blocks. Describe it clearly enough so that someone else could build it without seeing it. (Blocks may represent two-dimensional shapes—i.e., pattern blocks—or three-dimensional— i.e., wooden geometric solids.)
2.4.B Collect, organize, represent, and interpret data in bar graphs or picture graphs, make observations about data, draw conclusions, and make predictions.	**2.4.B Collect, organize, represent, and interpret data in bar graphs and picture graphs.**
2.4.C Model, create, and describe multiplication situations in which sets of equal size are joined.	**2.4.C Model and describe multiplication situations in which sets of equal size are joined.**

	The *Comments* should include language that: – Makes it clear that this is a conceptual introduction to multiplication and that only small numbers should be used: the existing example is good.
2.4.D Model, create, and describe division situations in which sets are separated into equal parts.	**2.4.D Model and describe division situations in which sets are separated into equal parts.** The *Comments* should include language that: – Makes it clear that this is a conceptual introduction to division and that only small numbers should be used.
2.4.E Interpret a fraction as a number of equal parts of a whole or a set.	No change

2.5. Core Processes: *Reasoning, problem solving, and communication*
Students further develop the concept that doing mathematics involves solving problems and talking about what they did to solve those problems. Second-grade problems emphasize addition and subtraction with increasingly large numbers, measurement, and early concepts of multiplication and division. Students communicate their mathematical thinking and make increasingly more convincing mathematical arguments. Students participate in mathematical discussions involving questions like "How did you get that?"; "Why did you use that strategy?" and "Why is that true?" Students continue to build their mathematical vocabulary as they use correct mathematical language appropriate to grade two when discussing and refining solutions to problems.

Current	Edited *Expectation* and *Comment*
2.5.A Identify the question(s) asked in a problem and any other questions that need to be answered in order to solve the problem.	No change
2.5.B Identify the given information that can be used to solve a problem.	No change
2.5.C Recognize when additional information is required to solve a problem.	No change
2.5.D Select from a variety of problem-	No change

solving strategies and use one or more strategies to solve a problem.	
2.5.E Identify the answer(s) to the question(s) in a problem.	No change
2.5.F Describe how a problem was solved.	No change
2.5.G Determine whether a solution to a problem is reasonable.	No change

Grade 3

The introduction to this new Core Expectation on whole numbers, addition and subtraction needs to be written.

Current	Edited *Expectation* and *Comment*
3.4.A Read, write, compare, order, and represent numbers to at least 10,000 using numbers, words, and symbols.	**3.4.A Read, write, compare, order, and represent numbers to 10,000 using numbers, words, and symbols.**
3.4.B Round whole numbers through 10,000 to the nearest ten, hundred, and thousand.	No change
3.4.D Fluently add and subtract whole numbers accurately using the standard regrouping algorithms.	No change
3.4.E Solve contextual problems involving addition and subtraction of whole numbers and justify the solutions.	**3.4.E Solve word problems involving addition and subtraction of whole numbers and verify the solutions.**
3.4.F Estimate sums and differences to predict solutions to problems or determine	**3.4.F Estimate sums and differences to approximate solutions to problems and determine reasonableness of answers.**

reasonableness of answers.	
	3.4.G Mentally add and subtract two- and three-digit numbers. Language in the *Comments* **should** include: – Examples/descriptions of strategies including compensation.

3.1. Core Content: *Concepts of multiplication and division (Operations, Algebra)*
Students learn the meaning of multiplication and division and how these operations relate to each other. They begin to learn multiplication and division facts and how to multiply larger numbers. Students use what they are learning about multiplication and division to solve a variety of problems. With a solid understanding of these two key operations, students are prepared to formalize the procedures for multiplication and division in grades four and five.

Current	Edited *Expectation* and *Comment*
3.1.A Represent multiplication as joining equal groups of objects using words, numbers, pictures, physical objects, and equations, and translate among representations.	**3.1.A Represent multiplication as repeated addition, arrays, counting by multiples, numbers, and jumps on the number line and connect the representation to the related equation.** Additional language in *Comments*, **similar** to the following, **should** address: – Students should be familiar with using words, pictures, physical objects, and equations to represent multiplication. – Students need to be able to connect various representations of multiplication to the related multiplication equation. Not all representations connect to each other in sensible ways.
3.1.B Represent division as equal sharing and forming equal groups of objects using words, numbers, pictures, physical objects, and equations, and translate among representations.	**3.1.B Represent division as equal sharing, repeated subtraction, equal lengths on the number line, and forming equal groups of objects and connect the representation to the related equation.** Additional language in *Comments*, **similar** to the following, **should** address: – Students should be familiar with using words, pictures, physical objects, and equations to represent division. – Students need to be able to connect various representations of multiplication to the related multiplication equation. Not all representations connect to each other in sensible ways.

3.1.C Determine products, quotients, and missing factors using the inverse relationship between multiplication and division.	The *Comments* **should** include the following: – An example that illustrates a missing factor.
3.1.D Explain and use strategies and properties of operations for remembering basic multiplication and division facts.	**3.1.D Apply and explain strategies to compute multiplication facts to 10 X 10 and the related division facts.**
3.1.E Quickly recall at least those multiplication facts and related division facts that have a factor of 1, 2, 5, or 10.	**3.1.E Quickly recall those multiplication facts for which one factor is 1, 2, 5, or 10 and the related division facts.** Language **similar** to the following **should** be added to the *Comments*: – Many students will learn all of the math facts to 10 X 10 by the end of third grade and all should be given the opportunity to do so.
3.1.F Describe orally, pictorially, or in writing a contextual problem that corresponds to a given multiplication or division equation.	**3.1.F Create word problems that match multiplication or division equations.** Add language **similar** to the following to the *Comments*: – Describe orally, pictorially, or in writing a contextual problem that corresponds to a given multiplication or division equation.
3.1.G Multiply any number from 11 through 19 by a single-digit number using the distributive property and appropriate place value concepts.	**3.1.G Multiply any number from 11 through 19 by a single-digit number using the distributive property and place value concepts.**
3.1.H Solve single- or multi-step contextual problems that involve multiplication and division, and justify the solutions.	**3.1.H Solve single- and multi-step word problems that involve multiplication and division, and verify the solutions.**

3.2. Core Content: *Fraction concepts (Numbers, Algebra)*
Students learn about fractions and how they are used. Students deepen their understanding of fractions by comparing and ordering fractions and by representing them in different ways. With a solid knowledge of fractions as numbers, students are prepared to be successful when they add, subtract, multiply, and divide fractions to solve problems in later grades.

Current	Edited *Expectation* and *Comment*
3.2.A Represent fractions that have denominators ranging from 2 to 12 using physical objects, pictures, numbers, and words, and translate among representations.	**3.2.A Represent fractions that have denominators of 2, 3, 4, 5, 6, 8, 10, and 12 as parts of a whole, parts of a set, and points on the numbers line.** Additional language in *Comments*, **similar** to the following, **should** address: – Students should be familiar with using words, pictures, physical objects, and equations to represent parts of a whole, parts of a set, and points on the number line.
3.2.B Compare and order fractions that have denominators ranging from 2 to 12.	No change
3.2.C Represent equivalent fractions that have denominators ranging from 2 to 12, and translate among representations.	**3.2.C Represent and identify equivalent fractions with denominators of 2, 3, 4, 5, 6, 8, 10, and 12.**
3.2.D Solve single- and multi-step contextual problems involving comparing fractions, and justify the solutions.	**3.2.D Solve single- and multi-step word problems involving comparing fractions and verify the solutions.**

3.3. Core Content: *Geometry (Geometry/Measurement)*
Students learn about lines and use lines and line segments in various ways with triangles and quadrilaterals. Understanding these basic shapes enables students to work with more complex shapes that can be broken down into triangles and quadrilaterals. Students connect this geometric work to numbers, operations, and measurement as they determine simple perimeters in ways they will remember when they later calculate perimeters of more complex figures in grades four and six.

This needs to be rewritten to reflect the revised standards

Current	Edited *Expectation* and *Comment*
3.3.A Identify and sketch parallel, intersecting, and perpendicular lines and line segments.	**No change**

3.3.B Identify and sketch right angles	No change
3.3.C Compare figures to determine congruence using geometric transformations (motions), such as reflections (flips), rotations (turns), and translations (slides).	~~3.3.C Compare figures to determine congruence using geometric transformations (motions), such as reflections (flips), rotations (turns), and translations (slides).~~ Moved to grade 4
3.3.D Identify, describe, and classify different types of quadrilaterals based on their properties.	**3.3.D Identify and describe the quadrilaterals: squares, rectangles, parallelograms, rhombi, trapezoids and kits.** Language in Comments should be revised to reflect the changed Expectation.
3.3.E Determine the number and location of lines of symmetry in a given figure.	~~3.3.E Determine the number and location of lines of symmetry in a given figure.~~ Moved to Grade 5.
3.3.F Identify, describe, and classify triangles as equilateral, isosceles, or scalene based on the number of congruent sides.	~~3.3.F Identify, describe, and classify triangles as equilateral, isosceles, or scalene based on the number of congruent sides.~~ Moved to Grade 5.
3.3.G Measure and calculate perimeters of two-dimensional figures.	~~3.3.G Measure and calculate perimeters of two-dimensional figures.~~ Moved to Grade 5.
3.3.H Solve single- and multi-step problems involving attributes of two-dimensional figures and justify solutions.	~~3.3.H Solve single- and multi-step problems involving attributes of two-dimensional figures and justify solutions.~~ Moved to Grade 5.

3.4. Additional Key Content *(Numbers, Operations, Algebra, Geometry/Measurement, Data/Statistics/Probability)*
Students solidify and formalize a number of important concepts and skills related to Core Content studied in previous grades. In particular, students extend critical concepts of the base ten number system to include large numbers, and they demonstrate their understanding of equivalence, both of which are important foundations for later work in algebra. They formalize procedures for adding and subtracting large numbers and apply these procedures in new contexts. Students also reinforce their knowledge of measurement as they use standard units for temperature, weight, and capacity.

Current	Edited *Expectation* and *Comment*
3.4.A Read, write, compare, order, and represent numbers to at least 10,000 using numbers, words, and symbols.	~~3.4.A Read, write, compare, order, and represent numbers to at least 10,000 using numbers, words, and symbols.~~ 3.4.A has moved into a new grade 3 Core Content area of whole numbers, addition and subtraction.
3.4.B Round whole numbers through 10,000 to the nearest ten, hundred, and thousand.	~~3.4.B Round whole numbers through 10,000 to the nearest ten, hundred, and thousand~~. 3.4.B has moved into in a new grade 3 Core Content area of whole numbers, addition and subtraction.
3.4.C Determine whether two expressions are equal and use "=" to denote equality.	No change
3.4.D Fluently add and subtract whole numbers accurately using the standard regrouping algorithms.	~~3.4.D Fluently add and subtract whole numbers accurately using the standard regrouping algorithms.~~ 3.4.D has moved into a new grade 3 Core Content area of whole numbers, addition and subtraction.
3.4.E Solve contextual problems involving addition and subtraction of whole numbers and justify the solutions.	~~3.4.E Solve contextual problems involving addition and subtraction of whole numbers and justify the solutions.~~ 3.4.E has moved into a new grade 3 Core Content area of whole numbers, addition and subtraction.
3.4.F Estimate sums and differences to predict solutions to problems or determine reasonableness of answers.	~~3.4.F Estimate sums and differences to predict solutions to problems or determine reasonableness of answers.~~ 3.4.F has moved into a new grade 3 Core Content area of whole numbers, addition and subtraction.
3.4.G Measure temperature in degrees Fahrenheit and degrees Celsius using a	No change

thermometer.	
3.4.H Estimate, measure, compare, and select appropriate-size units to express weight or mass using customary and metric units.	**3.4.H Estimate, measure and compare weight and mass using appropriate-sized U.S. customary and metric units.**
3.4.I Estimate, measure, compare, and select appropriate-size units to express capacity using customary and metric units.	**3.4.I Estimate, measure, and compare capacity using appropriate-sized U.S. customary and metric units.**
3.4.J Construct and analyze pictographs, frequency tables, line plots, and bar graphs.	No change

3.5. Core Processes: *Reasoning, problem solving, and communication*
Students in grade three solve problems that extend their understanding of core mathematical concepts—such as geometric figures, fraction concepts, and multiplication and division of whole numbers—as they make strategic decisions that bring them to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Current	Edited *Expectations* and *Comments*
3.5.A Determine the question(s) to be answered given a problem situation.	No change
3.5.B Identify information that is given in a problem and decide whether it is essential or extraneous to the solution of the problem.	**3.5.B Identify information that is given in a problem and decide whether it is necessary or unnecessary to the solution of the problem.**
3.5.C Identify missing information that is needed to solve a problem.	No change
3.5.D Determine whether a problem to	No change

be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	
3.5.E Select and use one or more appropriate strategies to solve the problem, and justify the selection.	**3.5.E Select and use one or more appropriate strategies to solve the problem.**
3.5.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.	No change
3.5.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.	No change
3.5.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	No change
3.5.I Summarize mathematical information, draw conclusions, and explain reasoning.	No change
3.5.J Make and test conjectures based on data (or information) collected from explorations and experiments.	No change

Grade 4

4.1. Core Content: *Multi-digit multiplication (Numbers, Operations, Algebra)*

Students learn basic multiplication facts and efficient procedures for multiplying two- and three digit numbers. They explore the relationship between multiplication and division as they learn related division and multiplication facts in the same fact family. These skills, along with mental math and estimation, allow students to solve problems that call for multiplication. Building on an understanding of how multiplication and division relate to each other, students prepare to learn efficient procedures for division, which will be developed in fifth grade. Multiplication of whole numbers is not only a basic skill, it is also closely connected to Core Content in grade four on area, and this connection reinforces understanding of both concepts. Multiplication is also central to students' study of many other topics in mathematics across the grades, including fractions, volume, and algebra.

Current	Edited *Expectations* and *Comments*
4.1.A Quickly recall multiplication and related division facts through 10 × 10.	**4.1.A Quickly recall multiplication facts through 10 X 10 and the related division facts.**
4.1.B Identify factors and multiples of a number.	No change
4.1.C Represent multiplication of two-digit by two digit numbers and describe how that representation connects to the related number sentence.	**4.1.C Represent multiplication of two-digit by two-digit number with place value models.**
4.1.D Multiply by 10, 100, and 1,000.	No change
4.1.E Compare the values represented by digits in whole numbers using place value.	No change
4.1.F Fluently multiply up to a three-digit number by one- and two-digit numbers accurately using the standard multiplication algorithm.	**4.1.F Fluently and accurately multiply up to a three-digit number by one- and two-digit numbers using the standard multiplication algorithm.** The *Comments* **should** be modified to: – *Include an example of a multiplication problem completed using the standard algorithm.* – Teachers should be aware that in some countries the algorithm might be recorded differently.
4.1.G Mentally multiply two-digit numbers by numbers through 10 and by multiples of 10.	No change
4.1.H Estimate products in order to predict results or determine reasonableness of answers.	**4.1.H Estimate products to approximate solutions to problems and determine reasonableness of answers.**

4.1.I Solve single- and multi-step contextual problems involving multi-digit multiplication, and justify the solutions.	**4.1.I Solve single- and multi-step word problems involving multi-digit multiplication and verify the solutions.**
4.1.J Solve contextual problems involving division, and justify the solutions.	**4.1.J Solve word problems involving division and verify the solutions.**

4.2. Core Content: *Fractions, decimals, and mixed numbers (Numbers, Algebra)*
Students solidify and extend their understanding of fractions (including mixed numbers) to include decimals and the relationships between fractions and decimals. Students work with common factors and common multiples as preparation for learning procedures for fraction operations in grades five and six. When they are comfortable with and knowledgeable about fractions, students are likely to be successful with the challenging skills of learning how to add, subtract, multiply, and divide fractions.

Current	Edited *Expectations* and *Comments*
4.2.A Represent decimals through hundredths using numbers, words, pictures, and physical objects, and translate among representations.	**4.2.A Represent decimals through hundredths with place value models, fraction equivalents and the number line.** Additional language in *Comments*, **similar** to the following, **should** address: – Students should be familiar with words, pictures, physical objects, and equations representing decimals through hundredths and be able to use at least two representations.
4.2.B Read, write, compare, and order decimals through hundredths.	No change
4.2.C Represent mixed numbers as fractions using numbers, pictures, and physical objects.	**4.2.C Convert a mixed number to a fraction and vice versa and visually represent the number.** Additional language in the *Comments* **should** address the following: – Students use words, numbers, pictures, and physical objects.
4.2.D Convert a decimal to a fraction	**4.2.D Convert a decimal to a fraction and vice versa, and visually represent the**

and vice versa, and visually represent the number using pictures or physical objects.	**number.** Additional language in *Comments*, **similar** to the following, **should** address: – Students should be familiar with using pictures and physical objects to visually represent the number.
4.2.E Compare and order decimals and fractions (including mixed numbers) on the number line, in number sentences, and in lists.	No change
4.2.F Write a fraction equivalent to a given fraction using common multiples.	No change
4.2.G Simplify fractions using common factors.	No change
4.2.H Round fractions and decimals to the nearest whole number.	No change
4.2.I Solve single- and multi-step contextual problems involving comparison of decimals and fractions (including mixed numbers), and justify the solutions.	**4.2.I Solve single- and multi-step word problems involving comparison of decimals and fractions (including mixed numbers), and verify the solutions.**

4.3. Core Content: *Concept of area (Geometry/Measurement, Algebra*

Students learn how to find the area of a rectangle as a basis for later work with areas of other two- and three-dimensional figures. They select appropriate units, tools, and strategies, including formulas, and use them to solve problems involving perimeter and area. Solving such problems helps students develop spatial skills, which are critical for dealing with geometric concepts. The study of area is closely connected to Core Content on multiplication, and connections between these concepts should be emphasized whenever possible.

Current	Edited *Expectations* and *Comments*
	4.3.A (Based on 3.3.C) Determine congruence of two-dimensional figures. Language in the *Comments* **should** develop the ideas below: – Students determine congruence making direct comparisons (i.e., "lift and place".) – Students describe the transformations of rectangles as flips, turns, and slides. Both the language and the concept of transformation will be more formally developed in grade 8.
	4.3.B (Based on 3.3.G) Measure and calculate the perimeter of a rectangle.
4.3.A Determine the approximate area of a two dimensional figure using square units.	**4.3.C (Based on 4.3.A) Determine the approximate area of a rectangle using square units.** The following **should** be added to the *Comments*: – A pictorial example using grid paper should be added.
4.3.B Determine the perimeters and areas of squares and other rectangles using formulas and explain why the formulas work.	**4.3.D. (Based on 4.3.B) Determine the perimeter and area of a rectangle using formulas and explain why the formulas work**. Additional language in the *Comments* **should** address the following: – Mention squares as special rectangles. – Remind teachers of the connection between area and multiplication and how useful is the area model of multiplication.
4.3.C Determine the areas of nonrectangular figures that can be composed or decomposed into rectangles.	**4.3.E (Formerly 4.3.C) Determine the areas of figures that can be broken down into rectangles.** The examples in the *Comments* need to be reviewed: 3 of the 4 are no longer relevant.
4.3.D Demonstrate that rectangles with the same area can have different perimeters, and that rectangles with the	4.3.F (Renumbered)

same perimeter can have different areas.	
4.3.E Solve single- and multi-step contextual problems involving perimeters and areas, and justify the solutions.	**4.3.G (Renumbered from 4.3.E) Solve single- and multi-step word problems involving perimeters and areas of rectangles, and verify the solutions.**

4.4. Additional Key Content *(Geometry/Measurement, Algebra, Data/Statistics/Probability)*
Students use coordinate grids to connect numbers to basic ideas in algebra and geometry. This connection between algebra and geometry runs throughout advanced mathematics and allows students to use tools from one branch of mathematics to solve problems related to another branch. Students also extend and reinforce their work with whole numbers and fractions to describe sets of data and find simple probabilities. Students combine measurement work with their developing ideas about multiplication and division as they do basic measurement conversions. They begin to use algebraic notation while solving problems in preparation for formalizing algebraic thinking in later grades.

Current	Edited *Expectations* and *Comments*
4.4.A Represent an unknown quantity in simple expressions, equations, and inequalities using letters, boxes, and other symbols.	No change
4.4.B Solve problems involving familiar unit conversions within either the customary or metric measurement system.	**4.4.B Solve problems involving familiar unit conversions, including time, within either the U.S. customary or metric measurement system.**
4.4.C Estimate and determine elapsed time using a calendar, a digital clock, and an analog clock.	No change
4.4.D Graph and identify points in the first quadrant of the coordinate plane using ordered pairs.	No change

4.4.E Determine the median, mode, and range of a set of data and describe what each measure indicates about the data.	No change
4.4.F Describe and compare the likelihood of event.	No change
4.4.G Determine a simple probability from a context that includes a picture.	No change
4.4.H Organize the results of probability experiments in clear displays and interpret the results.	**4.4.H Display the results of probability experiments in clear displays and interpret the results.**

4.5. Core Processes: *Reasoning, problem solving, and communication*
Students in grade four solve problems that extend their understanding of core mathematical concepts—such as multiplication of multi-digit numbers, area, and the relationships between fractions and decimals—as they make strategic decisions that bring them to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Current	Edited *Expectation* and *Comment*
4.5.A Determine the question(s) to be answered given a problem situation.	No change
4.5.B Identify information that is given in a problem and decide whether it is essential or extraneous to the solution of the problem.	No change

4.5.C Identify missing information that is needed to solve a problem.	No change
4.5.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	No change
4.5.E Select and use one or more appropriate strategies to solve the problem, and justify the selection.	**4.5.E Select and use one or more appropriate strategies to solve the problem, and explain why that strategy was choosen.**
4.5.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.	No change
4.5.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.	No change
4.5.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	No change
4.5.I Summarize mathematical information, draw conclusions, and explain reasoning.	No change
4.5.J Make and test conjectures based on data (or information) collected from explorations and experiments.	No change

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Grade 5

5.1. Core Content: *Multi-digit division (Operations, Algebra)*

Students learn efficient ways to divide whole numbers. They apply what they know about division to solve problems, using estimation and mental math skills to decide whether their results are reasonable. This emphasis on division gives students a complete set of tools for adding, subtracting, multiplying, and dividing whole numbers—basic skills for everyday life and further study of mathematics.

Current	Edited *Expectations* and *Comments*
5.1.A Visually represent multi-digit division and describe how that representation connects to the related number sentence.	**5.1.A Represent multi-digit division using place value models and connect the representation to the related number sentence.** Additional language in the *Comments* **should** address the following: – Students use pictures, grid paper to represent division and describe how that representation connects to the related number sentence.
5.1.B Determine quotients for multiples of 10 and 100 by applying knowledge of place value and properties of operations	No change
5.1.C Fluently divide numbers of up to four digits by one- and two-digit divisors accurately using the standard long division algorithm.	**5.1.C Fluently and accurately divide numbers of up to four digits by one- and two-digit divisors using the standard long division algorithm.** The *Comments* **should** be modified to: – *Include an example of a division problem completed using the standard algorithm.* – Teachers should be aware that in some countries the algorithm might be recorded differently.
5.1.D Estimate quotients in problems involving up to two-digit divisors to predict solutions to problems or determine reasonableness of answers.	**5.1.D Estimate quotients in word problems involving up to two-digit divisors to approximate solutions and determine reasonableness of answers.**
5.1.E Mentally divide two-digit numbers by one-digit divisors and explain the strategies used.	No change
5.1.F Solve single- and multi-step	**5.1.F Solve single- and multi-step word problems involving multi-digit division, and**

contextual problems involving multi-digit division, and justify the solutions.	**verify the solutions.** In the *Comments*, delete "as needed."
	5.1.G (New) Interpret fractions as division.

5.2. Core Content: *Addition and subtraction of fractions and decimals (Numbers, Operations, Algebra)*
Students use what they know about adding and subtracting whole numbers and build on their understanding of fractions and decimals as they learn procedures for adding and subtracting fractions and decimals. Students apply these procedures, along with mental math and estimation, to solve a wider range of problems that involve more of the types of numbers students see in other school subjects and in their lives.

Language similar to this should be incorporated. It is important that students see fractions and decimals as extensions of the whole number system and not as complexly separate objects.

Current	Edited *Expectations* and *Comments*
5.2.A Represent addition and subtraction of fractions and mixed numbers using words, numbers, pictures, and physical materials, and translate among representations.	**5.2.A Represent addition and subtraction of fractions and mixed numbers using visual and numerical models and connect the representation to the related number sentence.** Additional language in the *Comments* **should** address the following: – Students should be familiar with the use of pictures and physical materials to represent addition and subtraction of mixed numbers and be able to describe how that representation connects to the related number sentence. – If possible, a graphic representation of a number line would be a good example. 3/2 – 3/4 is a good visual.
5.2.B Represent addition and subtraction of decimals using words, numbers, pictures, and physical materials, and translate among representations.	**5.2.B Represent addition and subtraction of decimals using place value models and connect the representation to the related number sentence.** Additional language in the *Comments* **should** address the following: – Students should be familiar with using pictures and physical materials to represent addition and subtraction of decimals and be able to describe how that representation connects to the related number sentence.
5.2.C Given two fractions with unlike	Additional language in the *Comments* **should** address the following:

denominators, rewrite the fractions with a common denominator.	– When students are fluent in writing equivalent fractions it helps them compare fractions and helps prepare them to add and subtract fractions.
5.2.D Determine the greatest common factor and the least common multiple of two or more whole numbers.	No change
5.2.E Fluently add and subtract fractions, including mixed numbers.	Additional language in the *Comments* **should** address the following: – It is important to add whole numbers to fractions to help students understand what whole numbers are fractions and fractions are an extension of the whole numbers.
5.2.F Fluently add and subtract decimals.	Additional language in the *Comments* **should** address the following: – It is important to use decimals greats than 1 and also to include whole numbers to help students understand that whole numbers are decimals (5.0) and decimals are an extension of the whole numbers."
5.2.G Estimate sums and differences of fractions, mixed numbers, and decimals, to predict solutions to problems or determine reasonableness of answers.	**5.2.G Estimate sums and differences of fractions, mixed numbers, and decimals, to approximate solutions and determine reasonableness of answers.**
5.2.H Solve single- and multi-step contextual problems involving addition and subtraction of whole numbers, fractions (including mixed numbers), and decimals, and justify their solutions.	**5.2.H Solve single- and multi-step word problems involving addition and subtraction of whole numbers, fractions (including mixed numbers), and decimals, and verify their solutions.** Delete "as needed."

5.3. Core Content: *Concepts of volume and surface area (Geometry/Measurement, Algebra)*
Students learn how to determine volumes of rectangular prisms. They apply their knowledge of area measurement to determine surface areas of rectangular prisms. Students select appropriate units, tools, and strategies, including formulas, and use them to solve problems that involve length, area, and volume. These geometric and measurement skills are among the mathematics skills most applicable to situations outside of school, and they are useful in other school subjects such as science, art, and technical or design courses. Using formulas in geometric measurement develops another important connection among the branches of mathematics. **REWRITE TO REFLECT CHANGES**

Current	Edited *Expectations* and *Comments*
5.3.A Describe and classify polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.	~~5.3.A Describe and classify polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.~~ Move to grade 6 (6.4.G)
	5.3.A (New) Classify quadrilaterals. The *Comments* **should** include ideas **similar** to the following: – Students classify pictures and objects shaped like common quadrilaterals including parallelograms, kites, squares, rhombi, rectangles
~~5.3.B Construct and compare three-dimensional figures and their two-dimensional representations~~	~~5.3.B Construct and compare three-dimensional figures and their two-dimensional representations.~~ Move to grade 6
	5.3.B (New) Identify, sketch, and measure acute, right, and obtuse angles.
	5.3.C (New) Identify, describe, and classify triangles by angle measure and number of congruent sides. Language in the *Comments* **should** include: – The vocabulary used to classify triangles by their angle size: acute, right, obtuse. – The vocabulary used to classify triangle by the length of their sides: scalene, isosceles, equilateral.
5.3.C Determine the surface area of rectangular prisms.	~~5.3.C Determine the surface area of rectangular prisms.~~ Moved to 6.4.E
5.3.D Determine the volume of rectangular prisms using cubes	~~5.3.D Determine the volume of rectangular prisms using cubes.~~

	Moved to grade 6
5.3.E Determine the volume of rectangular prisms using a formula, and explain why the formula works.	~~5.3.E Determine the volume of rectangular prisms using a formula, and explain why the formula works.~~ To grade 6
	5.3.D (New *Expectation)* Determine the formula for the area of a parallelogram by relating it to the area of the rectangle. Language in the *Comments* about the following **should** be included: – A graphic is needed for this one.
5.3.F Solve single- and multi-step contextual problems involving three-dimensional figures, and justify the solutions.	~~5.3.F Solve single- and multi-step contextual problems involving three-dimensional figures, and justify the solutions~~. Moved to other grades
	5.3.E (New) Determine the formula for the area of a triangle by relating it to the area of a parallelogram. Language in the *Comments* about the following **should** be included: – A graphic is needed for this one – the graphic from 6.4.B does not work.
	5.3.F (Formerly 6.4.B) Determine the perimeters and areas of triangles and parallelograms. Language in the *Comments* about the following **should** be included: – The actual measurement might be given, so student computes – Student might measure, which introduces the approximate nature of measure. Figures need to be carefully selected.
Moved from 6.4D	**5.3.G (Formerly 6.4.D) Draw quadrilaterals and triangles from given information about sides and angles.** Language in the *Comments* in original *Expectation* is still appropriate.
Came from 3.3.E	**5.3.H (Related to 3.3.E) Determine the number and location of lines of symmetry in**

	triangles and quadrilaterals. Language in the *Comments* in original *Expectation* is still appropriate.

5.4. Core Content: *Representations of algebraic relationships (Operations, Geometry/Measurement, Algebra)*
Students continue their development of algebraic thinking as they move toward more in-depth study of algebra in middle school. They use variables to write simple algebraic expressions describing patterns or solutions to problems. They use what they have learned about numbers and operations to evaluate simple algebraic expressions and to solve simple equations. Students make tables and graphs from linear equations to strengthen their understanding of algebraic relationships and to see the mathematical connections between algebra and geometry. These foundational algebraic skills allow students to see where mathematics, including algebra, can be used in real situations and prepare students for success in future grades.

Current	Edited *Expectations* and *Comments*
5.4.A Describe, extend, and create numeric and geometric patterns.	**5.4.A Describe, extend, and create a rule for numeric and geometric patterns.** The following changes in the language in the *Comments* **must** be made: – "Describe what size and shape the next tower could be and explain why you think your answer is right." to "Determine a rule for the shape of the towers and number of cubes needed to extend this pattern."
5.4.B Write a rule to describe the relationship between two sets of data.	**5.4.B Given two sets of data that are linearly related, write a rule to describe the relationship.**
5.4.C Write simple algebraic expressions and evaluate them using substitution.	**5.4.C Write algebraic expressions that represent simple situations and evaluate the expressions using substitution**.
5.4.D Graph ordered pairs on the coordinate plane for two sets of data related by a linear rule and draw the line they determine.	**5.4.D Graph ordered pairs on the coordinate plane for two sets of data related by a linear rule and draw the line they determine**.
5.4.E Write and evaluate expressions involving parentheses.	**5.4.C Write algebraic expressions involving parentheses that represent simple situations and evaluate the expressions."**

5.5. Additional Key Content *(Numbers, Geometry/Measurement, Data/Statistics/Probability)*
Students extend their work with common factors and common multiples as they deal with prime numbers. Students extend and reinforce their use of numbers, operations, and graphing to describe and compare data sets for increasingly complex situations they may encounter in other school subjects and in their lives. **Students learn more about angles and triangles in preparation for future work in geometry. Delete this last sentence because of changes in this section.**

Current	Edited *Expectations* and *Comments*
5.5.A Classify numbers as prime or composite.	No change
5.5.B Identify, sketch, and measure acute, right, and obtuse angles.	~~5.5.B Identify, sketch, and measure acute, right, and obtuse angles~~. To Geometry section
5.5.C Classify triangles as acute, right, or obtuse based on their angle measures.	~~5.5.C Classify triangles as acute, right, or obtuse based on their angle measures.~~ Moved to Geometry section
5.5.D Determine and interpret the mean of a small-sized data set of whole numbers.	The language in the *Comments* **should** include the following: – Another example that shows the effect of an outlier on a data set. Each of the data points in the current set are reasonably close to the mean. It does not illustrate the *Expectation*.
5.5.E Construct and analyze line graphs.	**5.5.E. Construct and answer questions about line graphs.**

5.6. Core Processes: *Reasoning, problem solving, and communication*
Students in grade five solve problems that extend their understanding of core mathematical concepts—such as division of multi-digit numbers, surface area and volume of rectangular prisms, addition and subtraction of fractions and decimals, and use of variables in expressions and equations—as they make strategic decisions leading to reasonable solutions. Students use pictures, symbols, or mathematical language to explain the reasoning behind their decisions and solutions. They further develop their problem-solving skills by making generalizations about the processes used and applying these generalizations to similar problem situations. These critical reasoning, problem-solving, and communication skills represent the kind of mathematical thinking that equips students to use the mathematics they know to solve a growing range of useful and important problems and to make decisions based on quantitative information.

Current	Edited *Expectations* and *Comments*
5.6.A Determine the question(s) to be answered given a problem situation.	No change
5.6.B Identify information that is given in a problem and decide whether it is essential or extraneous to the solution of the problem.	No change
5.6.C Identify missing information that is needed to solve a problem.	5.6.C Determine whether additional information is needed to solve the problem.
5.6.D Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	No change
5.6.E Select and use one or more appropriate strategies to solve the problem, and justify the selection.	**5.6.E Select and use one or more appropriate strategies to solve the problem, and explain the choice of strategy.**
5.6.F Represent a problem situation using words, numbers, pictures, physical objects, or symbols.	No change
5.6.G Explain why a specific problem-solving strategy or procedure was used to determine a solution.	No change
5.6.H Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	No change

5.6.I Summarize mathematical information, draw conclusions, and explain reasoning.	No change
5.6.J Make and test conjectures based on data (or information) collected from explorations and experiments.	No change

Grade 6

6.1. Core Content: *Multiplication and division of fractions and decimals (Numbers, Operations, Algebra)*
Students have done extensive work with fractions and decimals in previous grades and are now prepared to learn how to multiply and divide fractions and decimals with understanding. They can solve a wide variety of problems that involve the numbers they see every day—whole numbers, fractions, and decimals. By using approximations of fractions and decimals, students estimate computations and verify that their answers make sense.

Current	Edited *Expectations* and *Comments*
6.1.A Compare and order non-negative fractions, decimals, and integers using the number line, lists, and the symbols <, >, or =.	No change
6.1.B Represent multiplication and division of non-negative fractions and decimals using numbers, pictures, or physical objects, and translate among representations.	**6.1.B Represent multiplication and division of non-negative fractions and decimals using area models and the number line and connect this operation to the related number sentence.** Additional language in *Comments*, **similar** to the following, **should** address: – Students should be familiar with grid paper, physical objects, and equations representing decimals through hundredths and be able to use at least two representations.
6.1.C Use convenient approximations of fractions and decimals to estimate products and quotients.	**6.1.C Estimate products and quotients of fractions and decimals.**
6.1.D Fluently multiply and divide non-negative fractions.	**6.1.D Fluently multiply and divide non-negative fractions and explain the inverse relationship between multiplication and division**.

	Language **similar** to the following should be added to the *Comments:* – It is important to include whole numbers to show that fractions form an extension of the whole number system.
	6.1.E Multiply and divide whole numbers and decimals by 100, 10, 1, 0.1, and 0.01. Language **similar** to the following should be added to the *Comments:* – This standard helps make it clear that the place value system for decimals and whole numbers is the same.
6.1.E Fluently multiply and divide non-negative decimals.	6.1.F (Renumbered, but unchanged) Language **similar** to the following should be added to the *Comments:* – It is important to include whole numbers to show that fractions form an extension of the whole number system.
6.1.F Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.	6.1.G (Renumbered, but unchanged)
6.1.G Solve contextual problems involving operations with fractions or decimals, and justify the solutions.	**6.1.H (Renumbered) Solve word problems involving operations with fractions or decimals, and verify the solutions.** .

6.2. Core Content: *Mathematical expressions and equations*
(Operations, Geometry/Measurement, Algebra)
Students continue to develop their understanding of how letters are used to represent numbers in mathematics—an important foundation for algebraic thinking. Students use tables, words, numbers, graphs, and equations to describe simple linear relationships. They write and valuate expressions and write and solve equations. By developing these algebraic skills at the middle school level, students will be able to make a smooth transition to high school mathematics.

Current	Edited *Expectations* and *Comments*

6.2.A Write mathematical expressions or equations with variables to represent information in tables or contextual situations.	**6.2.A Write mathematical expressions or equations with variables to represent information in tables and word problems.**
6.2.B Draw a first-quadrant graph on the coordinate plane to represent information in a table or contextual situation.	**6.2.B Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.**
6.2.C Evaluate mathematical expressions when the value for each variable is given.	No change
6.2.D Apply the commutative, associative, and distributive properties, and use the order of operations to evaluate mathematical expressions.	No change
6.2.E Solve one-step equations using number sense, pictures, physical objects, or properties of equality.	**6.2.E Solve one-step equations and verify solutions.** Additional language in the *Comments* should include: – Students solve equations using number sense, physical objects like balance scales, or properties of equality.
6.2.F Write and use mathematical expressions or equations to solve contextual problems, and justify the solutions.	**6.2.F Solve word problems using mathematical expressions and equations and verify solutions.**

6.3. Core Content: *Ratios, rates, and percents*
(Numbers, Operations, Geometry/Measurement, Data/Statistics/Probability)
Students extend their knowledge of fractions to develop an understanding of what a ratio is and how it relates to a rate and a percent. Fractions, ratios, rates, and percents appear daily in the media and in everyday calculations like determining the sale price at a retail store or figuring out gas mileage. Students solve a variety of problems related to such situations. A solid understanding of ratios and rates is important for work involving proportional relationships in grade seven.

Current	Edited *Expectations* and *Comments*
6.3.A Identify and write ratios as comparisons of part-to-part and part-to-whole relationships, solve contextual problems involving ratios, and justify the solutions.	**6.3.A Identify and write ratios as comparisons of part-to-part and part-to-whole relationships, solve word problems involving ratios, and verify the solutions.**
6.3.B Write ratios to represent a variety of rates, solve contextual problems involving rates, and justify the solutions.	**6.3.B Write ratios to represent a variety of rates, solve word problems involving rates, and verify the solutions.**
6.3.C Represent percents in various forms, solve contextual problems involving percents, and justify the solutions.	**6.3.C Represent percents in various forms, word problems involving percents, and verify the solutions.**
6.3.D Translate among the fraction, decimal, and percent representations of a number.	**6.3.D Represent percents visually and numerically and convert between the fractional, decimal, and percent representations of a number.**
6.3.E Identify the ratio of the circumference to the diameter of a circle as the constant and recognize 22/7 and 3.14 as common approximations of π.	No change
6.3.F Determine the experimental probability of an event using data collected in an experiment.	**6.3.F Determine the experimental probability of a simple event using data collected in an experiment.**
6.3.G Determine the theoretical probability of an event and its complement and represent the probability as a fraction or decimal between 0 and 1 or as a percent	**No change**

between 0 and 100.	

6.4. Core Content: *Two-dimensional figures (Geometry/Measurement, Algebra)*
Students apply formulas to determine the area and perimeter or circumference of a variety of geometric shapes. As they learn about these important concepts, students can solve problems involving more complex figures than in earlier grades and use geometry to deal with a wider range of situations. These fundamental skills of geometry and measurement are increasingly called for in the workplace and they lead to a more formal study of geometry in high school.

This section needs to be rewritten to reflect the Geometry now in this section.

Current	Edited *Expectations* and *Comments*
6.4.A Explain informally why the area formulas for triangles, rectangles, parallelograms, trapezoids, and circles make sense.	~~6.4.A Explain informally why the area formulas for triangles, rectangles, parallelograms, trapezoids, and circles make~~. Moved to other grades
6.4.B Determine the perimeter and area of triangles, rectangles, parallelograms, and trapezoids.	~~6.4.B Determine the perimeter and area of triangles, rectangles, parallelograms, and trapezoids.~~ Split between grades 4 and 5
6.4.C Determine the circumference and area of circles.	6.4.A (Renumbered)
6.4.D Draw quadrilaterals and triangles from given information about sides and angles.	~~6.4.D Draw quadrilaterals and triangles from given information about sides and angles.~~ Moved to 5th to become 5.3.H
6.4.E Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.	**6.4.B (Renumbered)**
6.4.F Solve contextual problems involving the relationships among radius, diameter, circumference, and area of circles, and justify the solutions.	**6.4.C (Rewritten and renumbered) Solve word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions.**

	6.4.D (New—Formerly 7.3.A) Recognize and draw two-dimensional representations of three-dimensional figures. **In the *Comments* add an example of a net** should **be added.**
	6.4.E Determine the surface area and volume of rectangular prisms and explain why the formulas work.
	6.4.F (New) Determine the surface area of a pyramid.
	6.4.G (Formerly 5.3.A) Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.
6.4.G Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures	**Moved to 8th**

6.5. Additional Key Content *(Numbers, Operations)*
Students extend their mental math skills now that they have learned all of the operations—addition, subtraction, multiplication, and division—with whole numbers, fractions, and decimals. Students continue to expand their understanding of our number system as they are introduced to negative numbers for describing positions or quantities below zero. These numbers are a critical foundation for algebra, and students will learn how to add, subtract, multiply, and divide positive and negative numbers in seventh grade as further preparation for algebraic study.

Current	Edited *Expectations* and *Comments*
6.5.A Develop and use strategies for mental computations with non-negative whole numbers, fractions, and decimals	No change
6.5.B Locate positive and negative integers on the number line and use integers to represent quantities in various contexts.	No change

6.5.C Compare and order positive and negative integers using the number line, lists, and the symbols <, >, or =.	No change

6.6. Core Processes: *Reasoning, problem solving, and communication*
Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade six, students solve problems that involve fractions and decimals as well as rates and ratios in preparation for studying proportional relationships and algebraic reasoning in grade seven. As they solve problems with two-dimensional geometric figures, students prepare for work involving three-dimensional figures in seventh grade.

Current	Edited *Expectations* and *Comments*
6.6.A Analyze a problem situation to determine the question(s) to be answered.	No change
6.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.	No change
6.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.	No change
6.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.	No change
6.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols an informal and formal mathematical language.	No change

6.6.F Apply a previously used problem-solving strategy in a new context.	No change
6.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning	No change
	6.6.H (New) Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.
6.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.	6.6.I (Renumbered)

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Grade 7

7.1. Core Content: *Rational numbers and linear equations (Numbers, Operations, Algebra)*
Students add, subtract, multiply, and divide rational numbers—fractions, decimals, and integers—including both positive and negative numbers. With the inclusion of negative numbers, students can move more deeply into algebraic content that involves the full set of rational numbers. They also develop the algebraic skill of solving equations that require more than one step, allowing them to approach problems that deal with a wider range of contexts than before. Using generalized algebraic skills and approaches, students can pursue a wide range of problems involving any type of rational number, adapting strategies for solving one problem to different problems in different settings with underlying similarities.

Current	Edited *Expectations* and *Comments*
7.1.A Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.	No change
7.1.B Represent addition, subtraction, multiplication, and division of positive	**7.1.B Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.**

and negative integers using numbers, physical objects, or pictures.	Additional language in the *Comments,* **similar** to the following, **should** be included: – Students should be familiar with the use of the number line and physical materials, such as colored chips, to represent computation with integers. They should connect numeric and physical representations to the computation.
7.1.C Add, subtract, multiply, and divide rational numbers.	**7.1.C Fluently add, subtract, multiply, and divide rational numbers.**
7.1.D Determine the absolute value of a number and recognize that it represents the distance of the number from zero.	**7.1.D Define and determine the absolute value of a number.** Additional language in the *Comments,* **similar** to the following **should** be included: – Students define absolute zero as the distance of the number from zero.
7.1.E Solve two-step linear equations.	No change
7.1.F Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.	~~7.1.F Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.~~ This is moving to the *Core Processes*. Students should routinely be given the opportunity to do this.
7.1.G Solve multi-step contextual problems involving rational numbers and justify the solutions.	**7.1.G Solve multi-step word problems involving rational numbers and verify the solutions.**

7.2. Core Content: *Proportionality and similarity (Operations, Geometry/Measurement, Algebra)*
Students extend their work with ratios to solve problems involving a variety of proportional relationships, such as making conversions between measurement units or finding the percent increase or decrease. They also solve problems involving the proportional relationships related to similar figures, and in so doing reinforce an important connection between numerical operations and geometric relationships. Students graph proportional relationships and identify the rate of change as the slope of the related line. The skills and concepts related to proportionality represent some of the most important connecting ideas across K–12 mathematics. With a good understanding of how things grow proportionally, students can understand the linear relationships that are the basis for high school mathematics. If learned well, proportionality can open the door for success in much of secondary mathematics.

Current	Edited *Expectations* and *Comments*
7.2.A Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.	No change
7.2.B Solve problems involving proportional relationships and justify the solutions.	**7.2.B Solve problems involving proportional relationships and verify the solutions.** **Consider** adding the additional to the *Comments*: – More complex problems such as dividing 100 into proportional parts of (4:3:3) or determining the amounts in 1 and ½ times a recipe. – Delete "as needed" – Language should be "should" rather than "consider" and then "something similar"
7.2.C Solve problems involving similar figures.	**7.2.C Define similarity and solve problems involving similar triangles and rectangles.**
7.2.D Read and make scale drawings and solve problems related to scale.	**7.2.D Make scale drawings and solve problems related to scale.**
7.2.E Draw the image of a figure, dilated by a given scale factor and with the center of dilation at the origin, in the coordinate plane.	~~7.2.E Draw the image of a figure, dilated by a given scale factor and with the center of dilation at the origin, in the coordinate plane~~. Moved to grade 8.
7.2.F Represent proportional relationships using graphs, tables, or equations, and translate among the representations.	**7.2F (Rewritten and renumbered) Represent proportional relationships using graphs, tables, and equations, and make connections among the representations.**
7.2.G Determine the slope of a line corresponding to the graph of a proportional relationship.	**7.2.E (Rewritten and renumbered) Determine the slope of a line corresponding to the graph of a proportional relationship and relate this to similar triangles.**
7.2.H Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.	No change

7.2.I Determine whether or not a relationship is proportional and explain your reasoning.	No change
7.2.J Solve problems involving conversions within or between measurement systems and justify the solutions.	**7.2.J Solve problems involving conversions within or between measurement systems and verify the solutions.**

7.3. Core Content: *Surface area and volume (Geometry/Measurement)*
Students extend their understanding of area and perimeter to finding the surface area and volume of three-dimensional figures. They apply formulas and solve a range of problems involving three-dimensional objects, including problems people encounter in everyday life, in certain types of work, and in other school subjects. With a strong understanding of how to work with both two-dimensional and three-dimensional figures, students have an important base for the geometry they will study in high school.

Current	Edited *Expectations* and *Comments*
7.3.A Recognize and draw two-dimensional representations, including nets and perspective drawings, of three-dimensional figures.	~~7.3.A Recognize and draw two-dimensional representations, including nets and perspective drawings, of three-dimensional figures.~~ Moved to grade 6
	7.3.A (Based on 7.3.D) Determine the volume of pyramids and cones using formulas.
7.3.B Verify formulas for surface area and volume of prisms and cylinders using models or verbal reasoning.	**7.3.B Determine the surface area and volume of cylinders and explain why the formulas work.**
7.3.C Determine the surface area and volume of prisms and cylinders.	~~7.3.C Determine the surface area and volume of prisms and cylinders.~~
7.3.D Apply formulas to determine the volume of pyramids and cones and explain why the formulas are	~~7.3.D Apply formulas to determine the volume of pyramids and cones and explain why the formulas are reasonable.~~

reasonable.	Moved to 7.3.A above.
7.3.E Describe the effect that a change in scale factor on one attribute of a two- or three dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.	No change
7.3.F Solve contextual problems involving surface area or volume and justify the solutions.	**7.3.F Solve word problems involving surface area or volume and verify the solutions.**

7.4. Core Content: *Probability and data (Data/Statistics/Probability)*
Students apply their understanding of rational numbers and proportionality to concepts of probability. They begin to understand how probability is determined, and they make related predictions. Students revisit how to interpret data, now using sophisticated types of data graphs and thinking about the meaning of certain statistical measures. Statistics, including probability, is considered one of the most important and practical fields of study for making sense of quantitative information, and it plays an important part in secondary mathematics in the 21st century.

Current	Edited *Expectations* and *Comments*
7.4.A Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.	No change
7.4.B Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.	No change
7.4.C Describe a data set using measures of center (median, mean, and mode) and variability (maximum, minimum, and range) and evaluate the suitability and limitations of using each	Additional language in the *Comments,* **similar** to the following **should** be included: Students should construct data sets for a given mean, median, mode, or range in the classroom.

measure for different situations.	
7.4.D Construct data sets for a given mean, median, mode, or range.	~~7.4.D Construct data sets for a given mean, median, mode, or range.~~ Moved to the *Comments* section of 7.4.D
7.4.E Construct and interpret histograms, stem and-leaf plots, circle graphs, and box-and whisker plots.	7.4.E Construct and interpret histograms, stem and-leaf plots, circle graphs, ~~and box-and whisker plots.~~ Moved to 8th
7.4.F Evaluate different displays of the same data for effectiveness and bias and justify your evaluation.	7.4.F Evaluate different displays of the same data for effectiveness and bias and explain your thinking.

7.5. Additional Key Content *(Numbers, Geometry/Measurement, Algebra)*
Students extend their coordinate graphing skills to plotting points with both positive and negative coordinates on the coordinate plane. Using pairs of numbers to locate points is a necessary skill for reading maps and tables and a critical foundation for high school mathematics. Students further prepare for algebra by learning how to use exponents to write numbers in terms of their most basic (prime) factors.

Current	Edited *Expectations* and *Comments*
7.5.A Graph ordered pairs of rational numbers and determine the coordinates of a given point on the coordinate plane.	**7.5.A Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.**
7.5.B Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.	No change

7.6. Core Processes: *Reasoning, problem solving, and communication*
Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade seven, students

solve problems that involve positive and negative numbers and often involve proportional relationships. As students solve these types of problems, they build a strong foundation for the study of linear functions that will come in grade eight.

Current	Edited *Expectations* and *Comments*
7.6.A Analyze a problem situation to determine the question(s) to be answered.	No change
7.6.B Identify relevant, missing, and extraneous information related to the solution to a problem	No change
7.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem	No change
7.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.	No change
7.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	No change
7.6.F Apply a previously used problem-solving strategy in a new context.	No change
	7.6.G (New) Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.
7.6.G Extract and organize mathematical information from	7.6.H (No change, but renumbered)

symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	
7.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.	7.6.I (No change, but renumbered)

Grade 8

8.1. Core Content: *Linear functions and equations (Algebra)*

Students solve a variety of linear equations and inequalities. They build on their familiarity with proportional relationships and simple linear equations to work with a broader set of linear relationships and learn what functions are. They use graphs and other algebraic techniques to model applied problems with mathematical functions represented by graphs and other algebraic techniques. This Core Content area includes topics typically addressed in a high school algebra or first-year integrated math course, but here this content is expected of all middle school students in preparation for a rich high school mathematics program that goes beyond these basic algebraic ideas.

Current	Edited *Expectations* and *Comments*
8.1.A Solve one-variable linear equations.	No change
8.1.B Solve one- and two-step linear inequalities and graph the solutions on the number line.	No change
8.1.C Represent a linear function with a verbal description, table, graph, or symbolic expression, and translate among these representations.	**8.1.C Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.**
8.1.D Determine the slope and *y*-intercept of a linear function described by a symbolic expression, table, and graph.	No change
8.1.E Interpret the slope and *y*-intercept of the graph of a linear function representing a contextual situation.	No change
8.1.F Solve contextual problems involving linear functions and justify the solutions.	**8.1.F Solve word problems involving linear functions and verify the solutions.**
8.1.G Determine and justify whether a	In the *Comments,* the first example for 8.1.G **should** be deleted.

given verbal description, table, graph, or symbolic expression represents a linear relationship.	– There is not enough information to solve the problem.

8.2. Core Content: *Properties of geometric figures (Numbers, Geometry/Measurement)*
Students work with lines and angles, especially as they solve problems involving triangles. They use known relationships dealing with sides and angles of triangles to find unknown measures, connecting geometry and measurement in practical ways that will be useful well after high school. Since squares of numbers arise when using the Pythagorean Theorem, students work with squares and square roots, especially in problems with two- and three-dimensional figures. Using basic geometric theorems such as the Pythagorean Theorem, students get a preview of how geometric theorems are developed and applied in more formal settings, which they will study in high school.

Current	Edited *Expectations* and *Comments*
	8.2.A (Based on 6.4.G) Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.
8.2.A Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.	8.2.B (Renumbered, but unchanged)
8.2.B Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.	8.2.C (Renumbered, but unchanged)
8.2.C Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.	8.2.D (Renumbered, but unchanged)

8.2.D Quickly recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers.	8.2.E (Renumbered, but unchanged)
8.2.E Demonstrate the Pythagorean Theorem and its converse and apply them to solve problems.	8.2.F (Renumbered, but unchanged)
8.2.F Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.	8.2.G (Renumbered, but unchanged)

8.3. Core Content: *Summary and analysis of data sets (Algebra, Data/Statistics/Probability)*
Students build on their extensive experience organizing and interpreting data and begin to apply statistical principles to analyze statistical studies or short statistical statements, such as those they might encounter in the newspaper, on television, or on the Internet. They use mean, median, and mode to summarize and describe information, even when these measures may not be whole numbers. Students use their knowledge of linear functions to analyze trends in displays of data. They create displays for two sets of data in order to compare the two sets and draw conclusions. They expand their work with probability to deal with more complex situations than they have previously seen. These concepts of statistics and probability are important not only in students' lives, but also throughout the high school mathematics program.

Current	Edited *Expectations* and *Comments*
8.3.A Use measures of center (mean, median, and mode) and variability (range and interquartile range) to summarize and compare data sets, and explain the influence of outliers on each measure.	**8.3.A Summarize and compare data sets in terms of variability and measures of center.** Language in the *Comments,* **similar** to the following, should address: – Students use mean, median, mode, range and interquartile range to summarize and compare data sets, and explain the influence of outliers on each measure.
8.3.B Select, construct, and analyze data displays to compare two sets of data and draw conclusions from the data.	**8.3.B Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.** Language in the *Comments,* **similar** to the following, should address: – Box-and-whisker plots are a particular focus.

8.3.C Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.	No change
8.3.D Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.	Language in the *Comments,* **similar** to the following, should address: – The strengths and weakness of random, census, convenience, and representative sampling should be included.
8.3.E Evaluate conclusions of statistical studies reported in the media in terms of the methods used and possible sources of bias.	**8.3.D Determine whether conclusions of statistical studies reported in the media are reasonable.**
8.3.F Determine probabilities for mutually exclusive, dependent, and independent events from small sample spaces.	No change
8.3.G Solve problems using counting techniques and Venn diagrams, and justify the solutions.	**8.3.G Solve problems using counting techniques and Venn diagrams, and verify the solutions.**

8.4. Additional Key Content *(Numbers, Operations)*
Students deal with a few key topics about numbers as they prepare to shift to higher-level mathematics in high school. First, they use scientific notation to represent very large and very small numbers, especially as these numbers are used in technological fields and in everyday tools like calculators or personal computers. Scientific notation has become increasingly important as "extreme units" continue to be identified to represent increasingly tiny or immense measures arising in technological fields. A second important numerical skill involves using exponents in expressions containing both numbers and variables, thus extending students' work with order of operations to include more complicated expressions they might encounter in high school mathematics. Finally, to help students understand the full breadth of the real number system, students are introduced to simple irrational numbers, thus preparing them to study higher-level mathematics in which properties and procedures are generalized for the entire set of real numbers.

Current	Edited *Expectations* and *Comments*
8.4.A Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.	No change
8.4.B Apply scientific notation to solve contextual problems involving very large or very small units.	No change
8.4.C Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.	No change
8.4.D Identify rational and irrational numbers.	No change

8.5. Core Processes: *Reasoning, problem solving, and communication*
Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade eight, students solve problems that involve proportional relationships and linear relationships, including applications found in many contexts outside of school. These problems dealing with proportionality continue to be important in many applied contexts, and they lead directly to the study of algebra. Students also begin to deal with informal proofs for theorems that will be proven more formally in high school.

Current	Edited *Expectations* and *Comments*
8.5.A Analyze a problem situation to determine the question(s) to be answered.	No change
8.5.B Identify relevant, missing, and extraneous information related to the solution to a problem.	No change
8.5.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.	No change
8.5.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.	No change
8.5.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	No change
8.5.F Apply a previously used problem-solving strategy in a new context.	No change
8.5.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	No change
	8.6.H (New) Write an equation that corresponds to a given problem situation,

	and describe a problem situation that corresponds to a given equation.
8.5.H Make and test conjectures based on data (or information) collected from explorations and experiments.	8.5.I (Renumbered, but unchanged)