Washington State Board of Education

Study on Mathematical Soundness of Four Selected High School Programs

Presentation by Strategic Teaching

March 12, 2009

Project History

- High school mathematics standards approved July 30, 2008
- OSPI reviewed mathematical programs
- OSPI review programs for mathematical soundness
- OSPI made initial recommendations
  - Holt Algebra 1, Geometry, and Algebra 2
  - Discovering Algebra, Geometry, Advanced Algebra
  - Core-Plus Mathematics Courses 1, 2, and 3

ST's Task

- Forego duplication of content alignment

- Redeploy resources to
  - Add another reviewer for mathematical soundness
    - Guershon Harel
    - W. Stephen Wilson

  - Review OSPI recommendations plus an additional program
    - Glencoe McGraw-Hill Algebra I, Geometry, and Algebra II

ST's Topics

- Write and graph linear functions; translate between forms; solve problems that can be represented by linear functions, equations, and inequalities. {A1.4.B (M1.3.D); A1.1.B (M1.1.B)}

- Translate among forms of a quadratic function; graph and interpret the meaning of each form; solve problems that can be represented by quadratic functions, equations, and inequalities. {A2.3.A (M2.2.C); A2.1.C (M2.1.C)}

- Look at the development and proof of the triangle sum theorem. {G.3.A (M2.3.E)}

Mathematical Soundness

- Mathematical justification
  - *Are central theorems stated and proved?*
  - *Are solution methods to problems, conditions, and relations justified?*
- Symbolism and structure
  - *Does the program develop fluency with algebraic manipulations and reasoning in general terms?*
- Language
  - *Is the language used clear and accurate?*
- Assigned problems
  - *Does the text include a sufficiently large number of nontrivial, holistic problems?*
  - *Do mathematical concepts taught emerge from non-contrived problems?*

ST's Approach

- Reviewers worked independently
- Met in Washington DC upon completion of reviews
- Each reviewer wrote individual report available @

www.strategicteaching.com_washington_state_standards_.html

- *High School Mathematics Curriculum Study* synthesizes the reviews' work

ST's Findings Summary

	Linear functions	Quadratic functions	Triangle Sum Theorem
Holt Algebra I, Geometry, and Algebra II	✓	✓	✓
Discovering Algebra, Geometry, Advanced Algebra	–	–	–
Glencoe McGraw-Hill Algebra I, Geometry, and Algebra II	✓ and –	–	✓
Core-Plus Course I, II, and III	✓ and –	✓	–

+	Mathematically sound
✓	Mathematical soundness meets minimum standard
✓ and –	Reviewers gave different scores
–	Mathematically unsound

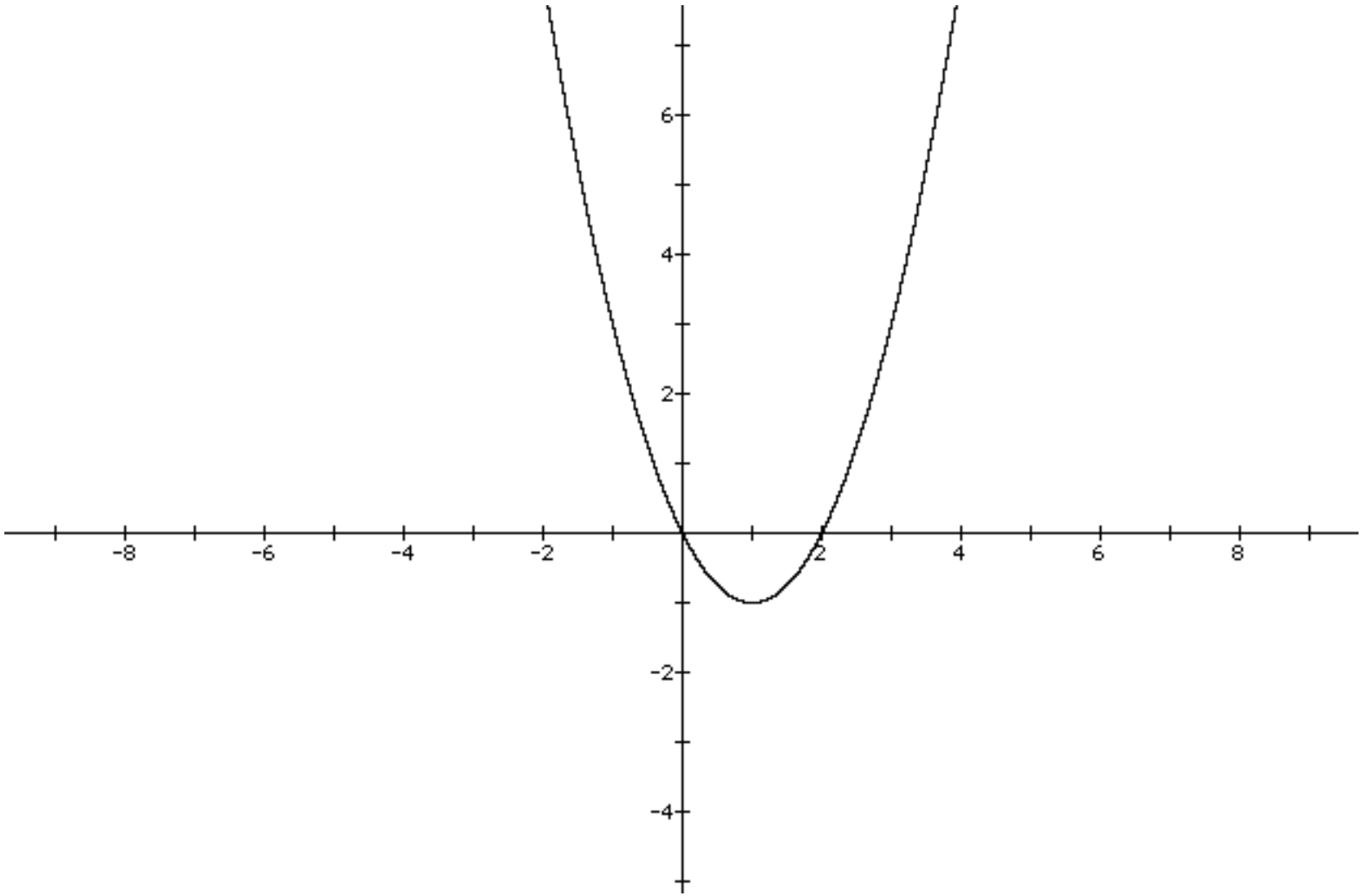
Holt Algebra 1, Geometry, Algebra 2

- Traditional organization; favors direct instruction and independent work with some hands-on labs; online support for homework
- All forms of linear and three forms of quadratic functions included.
- Excellent problems that require students to analyze the problem and determine how to solve it.
- Conceptual understanding takes a slight back seat to facts and procedures
- Algebraic facts and procedures are not deeply developed
  - General forms of linear functions rarely derived
  - Shallow development for the three forms of quadratic functions

Discovering Algebra, Geometry, and Advanced Algebra

- Content is traditionally organized; favors guiding students through problems in context,
- Presents all three forms of linear functions with the the point-slope form particularly well developed.
- Includes good problems for linear and quadratic functions and a proof of the triangle sum theorem
- Algebraic concepts and skills are not emphasized well enough
  - Content is presented in tiny pieces and not consolidated into the big ideas of mathematics
  - More a study of pictures of graphs than a study of quadratic functions
  - The idea that one method – graph, table, calculator, or algebraic approach – is superior to another in a specific situation is lost.

“PICTURE” OF GRAPH



Glencoe Algebra 1, Geometry, and Algebra 2

- Traditionally organized content; structured around mathematics rather than problems set in context; leans on directs instruction, but includes mini-labs; online support
- Contains well-crafted word problems
- Meticulous sequencing, including word problem strategies
- All aspects of symbolic manipulation are developed
- Mathematically careless
  - There are errors in some problems
  - Mathematically language is used incorrectly
  - There is a lack of attention to the difference between a postulate and a theorem in the proof of the triangle sum theorem.
- Mathematical ideas are presented as prescribed rules

Core-Plus Integrated Mathematics 1, 2, and 3

- Content integrated into four strands: algebra/functions, geometry/trigonometry, statistics/probability, discrete mathematics and spread over three years; students work in groups to solve problems that lead them to "discover" mathematics; designed for heterogonous grouping
- Develops a working understanding that a line in the plane is represented by a linear equation vise versa.
- All three forms of the quadratic formula and proof of quadratic formula are included.
- Symbolic manipulation is downplayed; tables, graphs, and calculators are emphasized.
  - The message is that the tools are interchangeable
  - The advantages of the general algebraic approach is lost. Shallow treatment of translating among forms of quadratic equation.
  - Proof of the triangle sum theorem depends on informally established definitions.

Suggestions for moving forward

- Recommend *Holt*. It exceeds the threshold for alignment with the content standards and meets the minimum standard for mathematical soundness.
- Do not recommend *Discovering* because it was found to be mathematically compromised within the scope of this project.
- Communicate to districts the additional challenges, identified within the scope of this project, that would come from the adoption of *Glencoe* and *Core-Plus*.
- Identify ways to strengthen the soundness of the programs meet minimum standards.
- Expand the examination of mathematical soundness to other programs with strong matches with respect to content standards

Suggestions for moving forward

- Consider ways to leverage district work
  - Form a statewide consortium or work groups to share supplements that shore up identified weaknesses
- Communicate the findings of this report and the more detailed reviewer reports to the publishers.
- Track student progress against curriculums adopted by districts.
- Establish a schedule to conduct a complete review of instructional programs every two years.
  - Consider a policy that recommends all programs that meet minimum thresholds for content and soundness.