Washington State Board of Education

Working to Raise Student Achievement Dramatically

MATH K-8 CURRICULAR MENU RECOMMENDATIONS

SUMMARY OF POLICY ISSUE/SBE STRATEGIC PLAN GOAL

Strategic Teaching, the Board's math consultant, will present its findings to the State Board of Education (SBE) on the four top ranked OSPI programs that best align with the new K-8 math standards. The Board will consider accepting the report and what action they wish to recommend to the Superintendent of Public Instruction. This work is related to the Board's goal of improving achievement for all students.

BACKGROUND

The legislature requires the Superintendent of Public Instruction to present to the SBE "no more than three basic mathematics curricular each for elementary, middle, and high school grade spans" (2SHB 2598 section 7 (a) from the 2008 legislative session) within six months of the adoption of the math standards. Within two months after the presentation of the recommended curricula, the SBE "shall provide official comment and recommendations to the Superintendent of Public Instruction regarding the recommended mathematics curricula. The Superintendent of Public Instruction shall make any changes based on the comments and recommendations from the State Board of Education and adopt the recommended curricula" (2SHB 2598 section 7 (b)). The Superintendent of Public Instruction (SPI) adopted the new K-8 math standards, after SBE approval, in April 2008. In the summer of 2008, the staff at the Office of Superintendent of Public Instruction hired Relevant Strategies to assist them with their instructional materials review of 25 math programs at the K-8 level. The OSPI solicited feedback from the SBE Math Panel at the beginning of its process, included several SBE Math Panel members in its review, and reconvened the SBE Math Panel to discuss their findings prior to release of their draft report. At that time OSPI identified the four programs at the elementary school level (grades K-5) and four programs at the middle school level (grades 6-8) that received the highest rankings.

Using a nationally competitive process, in August 2008, the SBE retained its consultant, Strategic Teaching, to assist with a review of the OSPI recommendations. SBE staff directed Strategic Teaching to review the top four programs for both elementary and middle school. The SBE staff, and its consultant, met with the Math Panel in late August 2008 to discuss the process that Strategic Teaching would use for its review.

At the September 2008 Board meeting, the Superintendent of Public Instruction recommended two programs each of the top four ranked programs for elementary grades K-5 (Bridges to Mathematics and Math Connects) and middle grades 6-8 (Holt, Math Connects).

Table 1: OSPI Initial Recommended Programs Selected from the Top Ranked Math Programs Best Aligned to the Revised K-8 Math Standards
Bold indicates OSPI initial recommendations

Elementary Level	Middle Level
Bridges in Mathematics	**Holt**
Math Connects	**Math Connects**
Investigations	Math Thematics
Math Expressions	Prentice Hall

The OSPI rationale for its recommendations for two elementary programs, Math Connects and Bridges in Mathematics, was that these were most closely aligned with state standards, Math Connects is fully available on line, these were the highest scoring programs overall, and both exceeded the minimum content threshold. The OSPI rationale for its recommendation of two middle programs, Holt Mathematics and Math Connects, was that Holt is a clear leader, based on analysis, the selection of Math Connects from the pool of tied programs gives districts a K-8 system, and both programs are fully available online.

Strategic Teaching will present its findings to the Board at the November 2008 Board meeting. Strategic Teaching shared an initial draft report with the SBE Math Panel in October 2008 to receive feedback. Strategic Teaching used a slightly different approach to the review of the top four programs, which is outlined in full in its report, which is also included in the Board's packet.

Some of the key differences included that Strategic Teaching:

- Reviewed fewer programs (8 vs. 25 for OSPI).
- Reviewed fewer grades for content match.
- Reviewed every lesson for content match.
- Spent more time on the review.
- Conducted a mathematical review across key topics.
- Used fewer reviewers (4 vs. 42 for OSPI).
- Used a different scoring rubric (4 point vs. 3 point used by OSPI).
- Reviewed content, but not other areas such as assessment, program organization and design, equity and access, etc. that OSPI also included.

Some of Strategic Teaching's findings were:

- OSPI's review process was rigorous and fair to publishers.
- OSPI should have used a process to examine mathematical soundness of key topic areas.
- OSPI and Strategic Teaching findings are similar enough to give Strategic

Teaching confidence that OSPI identified the programs with the best possible content match to Washington's standards.

- Strategic Teaching found that Math Expressions and Math Connects are the best programs for elementary school and Math Connects, Holt and Prentice Hall were the best programs for middle school.

Table 2: Strategic Teaching Final Recommended Programs Selected from the Top Ranked Math Programs Best Aligned to the Revised K-8 Math Standards
Bold indicates Strategic Teaching's final recommendations

Elementary Level	Middle Level
Math Connects	**Holt**
Math Expressions	**Math Connects**
Bridges in Mathematics	**Prentice Hall**
Investigations	Math Thematics

Strategic Teaching's findings support recommending Math Connects and Math Expressions at the elementary level (grades K-5). Strategic Teaching's rationale for the elementary programs was that Math Expressions is a rare find that includes the conceptual underpinnings and the procedural knowledge, with clear explanations between the two. Math Connects offers an acceptable choice because its shortcomings are easily remedied. Strategic Teaching found problems in the presentation of some of the mathematics core concepts in both Bridges and Investigations. While no program can be perfect for Washington, these two programs will need more support and supplementation.

Strategic Teaching's findings support recommending Holt, Math Connects, and Prentice Hall at the middle level (grades 6-8). Strategic Teaching's rationale for the middle programs was that Math Connects is the strongest program because it is better aligned to the state's standards then others reviewed. Math Connects will need only a small amount of supplementation. Holt and Prentice Hall are acceptable choices because they, too, include most of the content on the standards. Math Thematics is mathematically problematic in its treatment of proportionality.

The SBE Math Panel supported Strategic Teaching's findings. It appreciated the mathematical soundness review and expressed concern with the difficulty teachers may have trying to work with a variety of supplemental materials. They posed the following questions

- How will OSPI deal with Bridges to Mathematics based on Strategic Teaching's findings?
- How do we help districts do as well as they can when they have curricula that are not aligned?
- How do we think about doing future curricular reviews?
- How do we build incentives for publishers to align their curriculum with our standards?

POLICY CONSIDERATION

The Board needs to listen to the Strategic Teaching report and public feedback to determine what official comment and recommendations it wants to make to OSPI on the K-8 Math Curriculum.

PROPOSED ACTION

At this point, staff would recommend that the Board accept the Strategic Teaching report and recommend to OSPI:

1) Recommend for the elementary grade level (K-5): Math Connects and add Math Expressions on the elementary menu and revisit the decision on Bridges in Mathematics in light of information provided by Strategic Teaching in their final draft report to the Board.
2) Recommend for middle grade level (6-8): Holt and Math Connects and add Prentice Hall, in light of the additional information provided by Strategic Teaching in their final draft report to the Board.
3) Brief the Board, at the January SBE Board meeting, on the OSPI review for the supplemental K-8 materials, in particular what will be done for those programs that 70-80% of the schools are using now; findings from the district curriculum survey; actions OSPI plans to take to help districts with unaligned curriculum; and the results of the Request for Information for an online curriculum.
4) Brief the Board on suggestions for how to improve future curricular and instructional materials reviews at the SBE March 2009 meeting.
5) Brief the Board on how OSPI could provide incentives to publishers to align their curriculum with the new math standards.

Independent Study of Washington State

K-8 Curriculum Review

Final Report

October 27, 2008

Prepared by:

Strategic Teaching

Better learning by design

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Introduction

With the *Washington State K – 12 Mathematics Standards* firmly in place, Washington has turned its attention to finding instructional programs and materials that align with those standards. To a large degree, curriculum controls what is learned and how it is learned. Sound programs can help guide new teachers and can shore up the skills of teachers who lack a strong background in mathematics. Good curricula also provide top-notch teachers the materials they need to do their best work.

The Office of the Superintendent of Public Instruction, OSPI, was tasked by the legislature to identify programs for elementary and middle school. After a thorough curriculum review, OSPI presented its initial recommendations, *Math Connects* (K–5) and *Bridges in Mathematic* for elementary schools and *Holt Mathematics* and *Math Connects* (6–8) for middle schools, to the Washington State Board of Education, SBE, during the October SBE Board meeting.

The SBE recognizes the importance of choosing wisely since the mathematics programs that school districts implement drive instruction. With so much at stake, SBE requested that Strategic Teaching, ST, work with the SBE Math Panel to study the OSPI review. The purpose is to ensure that OSPI's highest-ranked programs offer students the best possible opportunity to meet the state's mathematics standards.

Executive Summary

During June of 2008, OSPI reviewed twelve elementary and thirteen middle school mathematics programs in order to determine which best align to Washington's standards. While no one expects that a single set of instructional materials will match perfectly to Washington's content, OSPI wants to identify those programs that come closest. OSPI will also identify supplemental programs that fill content gaps.

OSPI's curriculum review process involved four independent examinations of each program and a statistical analysis of the results. This produced two separate rank orderings of programs: one for grades K–5 and one for grades 6–8. The final report also includes detailed information for every program that was examined, down to the standard level.

At the request of the SBE, with guidance from the SBE Math Panel, ST reviewed OSPI's work. ST's work included four components. Specifically, ST:

- Reviewed the approach used by OSPI;
- Judged the degree to which the content in OSPI's top-ranked programs match the Washington's standards and compared its findings to those of OSPI;
- Provided an analysis, by an internationally recognized mathematician, of core mathematics topics in each program to determine the mathematical soundness of each program; and
- Summarized the key characteristics of each program.

ST found the approach OSPI used to review curriculum rigorous and similar in many ways to those used in most jurisdictions. OSPI's contractor, Relevant Strategies, was professional and thorough at each stage of the process. There are other methods of statistical analysis that might have been employed, but there is no reason to believe the outcomes would have been different. Most importantly, the curriculum review was designed to be fair to all publishers.

ST reviewed *Bridges in Mathematics*, *Investigations*, *Math Connects*, and *Math Expressions* for elementary school. *Holt Mathematics*, *Math Connects*, *Math Thematics*, and *Prentice Hall Mathematics* were reviewed at the middle school level. These are OSPI's highest-scoring programs. Other programs, such as the *Connected Math Project* that is widely used in Washington schools, were not reviewed because they did not meet OSPI's minimum threshold for content.

The comparison between OSPI's results and ST's results regarding how well the content in particular programs match to the state's standards is complex. ST's average scores were as high or higher than OSPI's average scores in all but two programs. However, there is a great deal of variation between the average scores of the two groups at the standard level and even at the topic level.

Many factors contribute to the variation in scores. OSPI and ST used rubrics with different numbers of points, OSPI conducted twice as many independent reviews of each program as did ST, and ST didn't give credit for off-grade level content. Also, ST looked at every lesson in every program while OSPI looked at the lessons identified in the publisher's alignment. This means ST looked at more material for some standards, causing higher scores.

Another important consideration is that rubric-based scoring involves applying professional judgment. The reason one has multiple reviewers is that reasonable people can look at the same materials and draw different conclusions about sufficiency of practice for a standard, for example.

ST believes that all of these factors contributed, in varying degrees, to the scores for each standard. Still, they converge meaning that ST's results verify OSPI's work.

ST included a mathematician's review of core topics—whole number multiplication, area of triangle, addition, subtraction, multiplication, and division with fractions and proportionality—to see how each is developed across grade levels in the programs under review. This is an examination of whether or not the instructional materials accurately represent the field of mathematics and whether or not errors related to the topics exist. To be clear, a mathematician is someone who studies the field of mathematics and is not an expert on teaching mathematics to students or on preparing teachers to do so.

ST's findings support using *Math Expressions* and *Math Connects* at the elementary level.

ST determined *Math Expressions* to be a rare find because it includes the conceptual underpinnings and the procedural knowledge, with clear explanations between the two, for each of the core topics examined by the mathematician. *Math Connects* offers an acceptable choice, because its few shortcomings are easily remedied.

ST found problems in the presentation of some of the mathematics in both *Bridges* and *Investigations*. While no program can be expected to be a perfect fit for Washington, these programs will need more support and supplementation than the other two programs under consideration.

ST's findings support using *Math Connects*, *Holt Mathematics*, and *Prentice Hall Mathematics* at the middle school level.

At the middle school level, *Math Connects* is the strongest program, because it is better aligned to the state's standards than the other reviewed programs and because it needs only a small amount of supplementation to be mathematically complete for the examined topics. *Holt Mathematics* and *Prentice Hall Mathematics* are acceptable choices because they, too, include most of the content in the standards. Student success with these two programs is more dependent on knowledgeable teachers to make connections among mathematical topics and to explain why procedures work.

Math Thematics, which has many characteristics ST liked, is mathematically problematic in its treatment of proportionality.

Charge to Strategic Teaching

SBE asked ST to do three tasks.

First, SBE asked ST to evaluate the process used by OSPI. A detailed discussion of OSPI's methods, including a comparison to those used by ST, begins on page 7 of this report. What is most important to know is that OSPI rigorously applied practices commonly used to review curriculum. The process afforded every program an equal opportunity to be selected.

Second, SBE asked ST to determine how well the content in OSPI's highest-ranking programs matches the content in Washington's standards in grades two, four, and seven. At the elementary school level, *Bridges in Mathematics*, *Investigations*, *Math Connects*, and *Math Expressions* were examined. At the middle school level *Holt Mathematics*, *Math Connects*, *Math Thematics*, and *Prentice Hall* were reviewed.

The relationship between the amount of matching content OSPI found in a program and the amount of matching content ST found in a matching program is complicated. In general the results converge, but scores for individual standards and for grade level topics often differ. Even when the total content for a program is similar, the content is often found in different standards.

More details about the comparison can be found in the sections for "Elementary School Findings" and "Middle School Findings."

Third, SBE requested that a mathematician review the programs to determine whether or not each is mathematically sound. To do this, ST identified core topics and examined their development across grade levels, from introduction to consolidation. ST looked at both the conceptual and the procedural sides of the topics, as well as how thoroughly procedures are explained. The topics of whole number multiplication, area of a triangle, arithmetic of fractions, and proportionality were examined.

Results can be found in the sections for "Elementary School Findings," "Middle School Findings," and "Individual Mathematics Programs." Organanizing the information this way does cause duplication but hopefully increases the report's usefullness to the reader.

In addition to SBE's charge, the SBE Math Panel expressed concerns for ST to investigate.

One concern was that OSPI's statistical analysis eliminated two programs at the elementary school level and two programs at the middle school level that should be included in ST's review. A strong case can be made that these

programs are statistical ties to programs that are included in the top four ranked programs. ST appreciates this concern, noting that ST would also have taken a different approach to the analysis. However, OSPI's methods are both sound and common, so ST was only tasked with the job of revising the four programs identified by OSPI.

Another concern centered around the weighting of some standards or strands. There was a discussion about some standards being more important than others and that important standards should be accorded more weight in the scoring process. Given the difficulty of finding consensus about which standards are most important, ST believes that reporting information by strand will at least provide information about where strong content matches exist.

ST looked at the differences in alignment scores in topics across grade levels to see if any troublesome patterns or inconsistencies appeared. This does not seem to be the case: there are no patterns to the discrepancies between OSPI and ST scores. The chart in Appendix C summarizes the differences by topics and grade levels.

The Panel also had questions about how OSPI and ST score standards that receive middle scores. A detailed discussion can be found in Appendix D, but broadly speaking, standards that receive a low total score from OSPI tend to receive a low total score from ST. Standards that receive a high total score from OSPI, tend to receive a high total score from ST.

One last point about the curriculum review should be noted. New programs and updated editions are published every year. Washington needs to establish a system of regularly and methodically considering new instructional materials for all subject areas.

Evaluation of OSPI's Curriculum Review

OSPI contracted with Relevant Strategies to conduct a review of all comprehensive elementary and middle school mathematics programs. Relevant Strategies applied the highest standards of professionalism to complete this task. It is clear that the review was designed to answer an open question, "Which sets of instructional materials meet the Washington State standards?" and not to support a foregone conclusion.

Meticulous attention to details throughout the process minimized the possibility that unconsidered factors would influence outcomes. For example, each publisher had the same amount of square footage to display its materials. Serious effort was made to eliminate or at least minimize bias from usual sources. In particular, each reviewer was asked if he or she could review a specific program fairly—without bias—before he or she was given that program to evaluate.

It is clear that the review was designed to answer an open question, "Which sets of instructional materials meet Washington's standards?" and not to support a foregone conclusion.

OSPI convened a group of forty-two people between June 22 and June 27, 2008, to review the instructional materials. During that time the reviewers received training on the standards and training related to scoring. The work was supported with daily meetings to discuss issues related to scoring and continuous on-site access to expertise. Almost every program had four reviews, but time constraints meant that one program at one grade level was reviewed by three, rather than four, reviewers.

Reviewers used a three-point rubric to match and score every standard for every program at every grade level, kindergarten through eight. The points on OSPI's rubric for content ranged from zero, meaning the standard is not met, to two, meaning the standard is fully met.

Reviewers who scored the same standard zero and two—a difference of two points—were given the opportunity to discuss their rationale and change their scores. The reviewers were systematically assigned and reassigned in ways that reduced the possibility of "scorer drift," the tendency of a group that works together for an extended period of time to begin to think and score alike in ways that may be different than the main group.

A total of 12 elementary programs and 13 middle school programs were evaluated.

OSPI scored and ranked the programs on multiple components. Specifically it examined the degree to which each program aligns with the content in *Washington's K – 8 Mathematics Standards* and 1) quality of the program's organization and design, 2) balance of student experience, 3) assessment, 4) instructional planning and professional support, and 5) equity and access.

The scores were used to compute scaled composite scores[1] that weighted content 70 percent of the overall score. There is some variation in the weights given to the other attributes, but the total contribution of the non-content factors is 30 percent. The average of all of the scaled composite scores of the standards across all of the grade levels produced a rank ordering of elementary school programs and another rank ordering of middle school programs.

Particularly commendable aspects of OSPI's review are 1) the amount of input and feedback from stakeholder groups, 2) the rigorous attention to detail in inventory, training, and scoring, and 3) the specificity of the results and their usefulness to school districts.

OSPI applied a particularly formal and systematic approach to its process—the type often associated with assessment events. Given the interest and tension around curriculum in Washington, the exceptional attention to detail was probably wise.

There is probably no best way for a jurisdiction to review curriculum. OSPI's methods are well within the typical range and meet the most critical criteria: the process was inclusive and it fairly considered all viable programs.

Particularly commendable aspects of OSPI's review are 1) the amount of input and feedback from stakeholder groups, 2) the rigorous attention to detail in inventory, training, and scoring, and 3) the specificity of the results and their usefulness to school districts.

All this said there are three areas that ST would have approached differently.

First, mathematicians should have been involved in reviewing the instructional materials. Content can be included but presented in ways that build long-term misunderstandings among students. While mathematicians do not necessarily understand the pedagogical issues related to teaching mathematics, they bring a knowledge of the discipline of mathematics that most mathematics educators lack.

Second, a four-point rubric has inherent advantages over a three-point rubric. The additional score point forces more refined decisions and the even number of levels forces the reviewer to make a decision about whether the standard is closer to "completely met" or "completely missing." Last, the

1 The score that is given divided by the maximum possible score, which converts the score to a scale of "1". This means a standard judged fully met all the points gets "1" no matter how many actual points are awarded and allows comparisons to be made because the scale is the same.

middle level in odd-numbered rubric tends to become a catchall and to receive a disproportionate number of scores.[2]

Third, there are two questions about the statistical analysis. To be clear, ST is not suggesting that the results would be different from those produced by OSPI because of the data analysis.

ST questions how well the data set stands up to the amount of analysis that has been done. The simplest example can be seen in the scaled score. This is the average of four scores that range from zero to two translated to a one-point scale. To take the scaled average of these professional judgments, opinions really, out beyond one or two decimal places gives a false impression about precision of the data.

To take the scaled average of these professional judgments, opinions really, out beyond one or two decimal places gives a false impression about the precision of the data.

ST also questions the method employed by OSPI to look at reviewer bias. OSPI chose to use t-test and adjust for multiple comparisons, which is a sound educational statistics method. ST believes that a more appropriate approach would have been to apply the generalizability theory, a methodology designed for situations exactly like curriculum review. That being said, OSPI's approach is able to identify programs that align to Washington's standards and other methods most likely would not have altered on these results.

2 Wiggins, Grant P. Educative Assessment: Designing Assessment to Inform and Improve Student Performance (San Francisco: Jossey-Bass, 1998), VI-5:1. Retrieved from Knowledge Quest on the Web on October 4, 2008 http://www.ala.org/ala/mgrps/divs/aasl/aaslpubsandjournals/kqweb/kqarchives/volume31/312philip.cfm#Ref6

Differences in Scoring Methods

There are differences—important differences—between the approaches of OPSI and ST that are largely driven by the differences in project goals. OSPI had the task of reviewing all viable comprehensive mathematics programs in order to recommend up to three programs for the elementary schools and up to three programs for middle schools. ST has the task of reviewing OSPI's work and so has only to sample and verify OSPI's results. These differences are summarized in the following chart.

	OSPI	ST
Reviewed Programs	12 Elementary 13 Middle	4 top-ranked elementary 4 top-ranked middle
Content Identification	Publisher's alignment is used to locate matching lessons. Reviewers examine lessons to determine if the content matches the standard. After content is scored, reviewers make overall judgments about the other attributes that are evaluated.	Every lesson is reviewed and the content in the lesson mapped to the standards. After the scoring is complete, the publisher's alignment is used to double check that all identified lessons are considered. Notes about the program are taken throughout the process and then used to discuss the non-scored attributes at the end.
Scored Attributes	Content Program organization and design Balance of student experience Assessment Instructional planning and professional support Equity and access	Content
Mathematician Review		ST mathematician, W. Stephen Wilson, Ph.D., reviews key topics and their development across grade levels. For this review, grade placement is not important
Non-Scored Analysis		Program organization and design Instructional approaches Presence of extra content in the program and not in the standards General program usability
Number of Reviews	4 independent reviews	2 independent reviews
Number of Reviewers	42	4*
Time Spent on Review	An average of 3.5 hours is spent per review.	This time was not carefully monitored, but reviewers report spending between 6 and 18 hours on a program at a grade level. The majority of the reviews, including discussion time between reviewers, range from 9 to 12 hours.
Non-matching Scores	Reviewers who have scores with 2-point spreads, discuss those differences and make changes based on their own discretion.	Reviewers discuss all non-matching scores (1-point spread) and make changes based on their own discretion.
Grade Levels	All grades, K – 8	2, 4, 7
Training	Extensive training on standards; training on rubric and review process; daily calibration meetings	Reviewers had intimate knowledge of standards; collaborative rubric development; calibration training using curriculum materials from project
Rubric	3-point rubric	4-point rubric

*Biographical information on ST reviewers can be found in Appendix E

There are some differences that cannot be mitigated. While there is the additional detail that naturally travels with additional score points in ST's rubric, there is one missing element that should be noted. OSPI considers content that is above or below grade level by a year to be partially meeting the standard and deserving of a "one" score. Since ST only reviewed selected grade levels and reviewers did not have access to off-grade materials, it did not do this. Such standards would receive a zero from ST. Appendix B offers a side-by-side comparison of the score points that match to each other on the OSPI and ST rubrics.

The effects of some of the differences in scoring approaches can be minimized. For example, as previously discussed, using scaled scores allows comparisons between three-point rubrics and four-point rubrics.

Focusing ST's work to look at OSPI's scores related to the content match also helps. This defines the work as an apples-to-apples comparison by creating a common core—content—that both groups examine. Fortunately, OSPI maintained separate data for its content scores.

In both reviews, trained and skilled adults apply a rubric to determine the degree to which the content in a standard is covered in a particular mathematics program.

The effects of some of the differences, such as the additional time spent by ST, are hard to determine. It seems reasonable there will be an effect, and that this effect will vary. Publishers should have identified the important content matches for OSPI reviewers to examine, but this is not the same as ST having unlimited time to examine every lesson in detail.

Some of the differences, such as the mathematician's review, will have no effect on the numerical scores. This is a separate analysis and the results are not co-mingled.

Despite these differences, the reviews are more similar than not. In both reviews, trained and skilled adults apply a rubric to determine the degree to which the content in a standard is covered in a particular mathematics program. Both OSPI and ST are interested in identifying programs that make it likely the typical student in an ordinary classroom will meet the standard.

While it is not reasonable to expect the numbers to match exactly, it is reasonable to expect ST's process to verify OSPI's results.

Elementary School Findings
Comparison of Content Alignment

The major purpose of this study is to determine whether or not ST agrees with OSPI's findings related to how well the content in selected mathematics programs aligns to Washington's standards.

In order to compare ST's results to those of OSPI, the average scaled score is computed for each standard for both groups. To do this all of the scores are added together and divided by the total possible points. This puts each group's average score on a scale of one, making comparison much easier.

For example, if four OSPI reviewers scored a particular standard 0-1-1-1, the scaled score would be 3 ÷ 8 = 0.38, since 8 is the maximum number of points possible based on the OSPI rubric that was used. Supposing that two ST reviewers gave this hypothetical standard the scores of 1-2, the corresponding scaled score would be 3 ÷ 6 = 0.50, since ST uses a rubric with three at the highest score. This allows the results to be compared on the same scale and shows a difference of 0.12.

There is danger in reading too much into these numbers, particularly at the standard level.

There is danger in reading too much into these numbers, particularly at the standard level. At first glance, in the example above, one might think that ST found a better content alignment in the particular standard than did OSPI. Actually, OSPI and ST agree that the program is missing content and/or key teaching strategies. The difference is due to the combination of OSPI having more reviews of the program and ST having more score points on its rubric. The point is, there will frequently be differences in scores, even when reviewers agree.

As scores are aggregated, the artificial differences related to scoring methodology should disappear. For that reason, ST is comparing results at the mathematical topic and at the grade level, rather than at the individual standard level.

In an attempt to make sense of the differences, ST is using the percent form. It is easier to talk and think about a 12 percent difference, than one of 0.12. Another word of caution is in order. Even with a literal interpretation, a difference of 12 percent means that ST found 12 percent more total alignment between the content and the program than OSPI. It does not mean ST found 12 percent more content alignment than did OSPI, which would be an increase of 0.12 over 0.38 or an increase in alignment of 32%.

A false sense of precision can be implied when averages are calculated to multiple decimal places. For this reason, ST will use only two decimal places in its tables.

Another danger associated with over analysis of this data is that rubric-based evaluation is not a precise science. Reasonable people, making every attempt to make fair decisions, can draw rational conclusions that differ.

All that said, quantifying the numbers offers precious insight into reviewer thinking and allows systematic evaluation and comparison that wouldn't otherwise be possible. With the limits of the methodology clearly in mind, ST will assume that differences at the mathematical topic and grade level represent real differences in the evaluation of content alignment, even if those differences are not perfectly captured by the numbers.

Grade 2	OSPI Scores by Section	ST Scores by Section	Difference*
Bridges in Mathematics			
2.1. Place value and the base ten system	0.88	0.67	-21%
2.2. Addition and subtraction	0.83	0.72	-11%
2.3. Measurement	0.90	0.83	-7%
2.4. Additional key content	0.78	0.27	-51%
2.5. Reasoning, problem solving, and communication	0.93	0.69	-24%
Bridges Grade 2 Average Scale Score	**0.86**	**0.65**	**-21%**
Investigations			
2.1. Place value and the base ten system	0.31	0.42	10%
2.2. Addition and subtraction	0.74	0.69	-5%
2.3. Measurement	0.68	0.57	-11%
2.4. Additional key content	0.65	0.77	12%
2.5. Reasoning, problem solving, and communication	0.63	0.69	7%
Investigations Grade 2 Average Scale Score	**0.61**	**0.63**	**2%**
Math Connects			
2.1. Place value and the base ten system	0.77	0.75	-2%
2.2. Addition and subtraction	0.74	0.70	-3%
2.3. Measurement	0.55	0.73	18%
2.4. Additional key content	0.80	0.93	13%
2.5. Reasoning, problem solving, and communication	0.89	0.83	-6%
Math Connects Grade 2 Average Scale Score	**0.76**	**0.78**	**2%**
Math Expressions			
2.1. Place value and the base ten system	0.56	0.81	24%
2.2. Addition and subtraction	0.79	0.85	6%
2.3. Measurement	0.58	0.73	16%
2.4. Additional key content	0.80	0.73	-7%
2.5. Reasoning, problem solving, and communication	0.80	0.81	1%
Math Expressions Grade 2 Average Scale Score	**0.69**	**0.80**	**11%**

* Apparent discrepencies in the difference are the result of rounding

Grade 4	OSPI Scores by Section	ST Scores by Section	Difference
Bridges in Mathematics			
4.1 Multi-digit Multiplication	0.58	0.88	31%
4.2 Fractions, decimals, and mixed numbers	0.57	0.89	32%
4.3 Concept of area	0.65	0.94	30%
4.4 Additional key content	0.72	0.79	7%
4.5 Reasoning, problem solving, and communication	0.61	0.88	27%
Bridges Grade 4 Average Scale Score	0.62	0.85	26%
Investigations			
4.1 Multi-digit Multiplication	0.68	0.58	-9%
4.2 Fractions, decimals, and mixed numbers	0.34	0.35	1%
4.3 Concept of area	0.44	0.61	17%
4.4 Additional key content	0.41	0.54	14%
4.5 Reasoning, problem solving, and communication	0.60	0.48	-12%
Investigations Grade 4 Average Scale Score	**0.50**	**0.51**	**1%**
Math Connects			
4.1 Multi-digit Multiplication	0.83	0.85	3%
4.2 Fractions, decimals, and mixed numbers	0.81	0.74	-6%
4.3 Concept of area	0.63	0.58	-4%
4.4 Additional key content	0.81	0.79	-2%
4.5 Reasoning, problem solving, and communication	0.69	0.77	8%
Math Connects Grade 4 Average Scale Score	**0.76**	**0.76**	**0%**
Math Expressions			
4.1 Multi-digit Multiplication	0.69	0.92	23%
4.2 Fractions, decimals, and mixed numbers	0.64	0.87	23%
4.3 Concept of area	0.60	0.97	37%
4.4 Additional key content	0.38	0.94	56%
4.5 Reasoning, problem solving, and communication	0.35	0.77	42%
Math Expressions Grade 4 Average Scale Score	**0.53**	**0.90**	**37%**

There is a lot to say about how ST's results compare to OSPI's results.

In a perfect world, there would be a more consistent relationship between ST's results and OSPI's results. Sometimes, as in grade 2 *Investigations* and *Math Connects* ST virtually matches OSPI's scores. Other times, as in grade 2 *Bridges*, ST scored a much weaker content match. And still other times ST finds the content alignment between a program and Washington's standards to be stronger than OSPI does.

There are reasons beyond those related to the number of reviews and score points on the rubric that could cause these differences. Some factors

—unlimited time, examination of every lesson—might cause ST's scores to be higher. There are other reasons—not considering off-grade level content, reviewer discussions about every different score point—that might cause ST to have lower scores. Apparently the effects of these variables fluctuate.

Grade two *Bridges*, illustrates both kinds of variability within the same program.

	OSPI	ST	Difference
Bridges Grade 2 Average Scale Score	0.86	0.65	-21%
Bridges Grade 4 Average Scale Score	0.62	0.88	26%

In Grade two, OSPI finds that 86 percent of the possible content is present, but that only 62 percent of the possible content is present in grade four. ST has almost opposite findings, seeing a much stronger alignment at grade four, but if one thinks about the aggregate of both grade levels, the differences almost disappear.

To shed light on the inconsistencies, ST looked first at grade two *Bridges*, where OSPI has higher scores and then at grade four *Expressions*, where ST has higher scores. ST then selected three standards from each program with disparate scale scores and looked at its reviewer's notes to better understand why those standards were given particular scores.

One of two situations seems likely:

- ST had the opportunity to look at more of the program material because it had the luxury of reviewing every lesson and taking as much time as needed.
- Different reviewers looked at the same material and drew different conclusions. Reasonable people, making every attempt to make fair decisions, can draw rational conclusions that differ.

2.4.B Collect, organize, represent, and interpret data in bar graphs and picture graphs.

(OSPI score = 1.00; ST score = 0.33)

ST Comments:

> While there is an emphasis on collecting data (Volume 2, Unit 5 and Volume 3, Unit 7), the other performances are also amply represented as they pertain to bar graphs although more emphasis on interpretation is needed. Probability is considered along with data analysis – at this age would be better to separate the two. Pie graphs are also introduced which may be too much at this age since they have little understanding of fractions. Did not see pictographs.

While not likely, it is possible that both ST reviewers overlooked lessons about pictographs that OSPI reviewers noticed. It is more likely that OSPI included lessons that should have been excluded because the *Bridges'* publishers identified them to be replaced with supplemental lessons designed especially for Washington. There were several lessons on "glyphs" that could easily have been reviewed if one didn't notice or remember to keep referencing *Bridges'* directions on which lessons should be replaced with which supplemental lessons.

Or it may be that OSPI have scored the content alignment higher because the reviewers felt "pictographs" were not the critical part of the standard. In this case it would be an example of reasonable people drawing different conclusions from the same evidence.

2.2.C Add and subtract two-digit numbers efficiently and acurately using a procedure that works with all two-digit numbers and explain why the procedure works.

(OSPI score = .75; ST score = 0.17)

ST Comments:

> The standard algorithm for addition and subtraction is shown in Supplement A5 with the caveat to "be sure they understand that it is not a replacement for all the strategies they have already invented; it is just another option," **(Sup A5.16 to A5.19).** While mastery of the standard algorithm is not expected in second grade, neither should it be a black sheep. Students share their mental strategies (counting on, making tens, etc.) but strategies for working with numerals on paper seems to be missing. There is no reason to believe that students are being led to use "efficient and accurate methods that work every time" beyond manipulatives. **Work Place 9** "Key Chain Charms" has students doing 2-digit subtract with renaming "using methods that make sense to them,"

(TE 631 to 632). **A.5-4** is about place value as applied in addition and subtraction. There are a few pages of practice, which are often homework (TE 753) but it is not clear what students are practicing. Publisher cites 7-9, which is a class discussion of mental strategies and 5-23 to 5-25, which depends on models.

While there is a good amount of practice for adding and subtracting two-digit numbers (teachers told to include every week) not clear that a procedure that "works with all two-digit numbers" is singled out or privileged nor that students are asked to explain why that procedure works. Standard algorithm that involves place value is included although student innovations are encouraged from beginning to end. In May, e.g., student-invented strategies are still being solicited in Activity 4, Set A5. All in all, there is too much emphasis on game playing and too much emphasis on methods other than the standard algorithm.

2.4.D Model and describe division situations in which sets are separated into equal parts.

(OSPI score = 0.63; ST score = 0.17)

ST Comments

A score of "1" is given because of the lesson at the very beginning of the year about ants dividing into columns. It is unusual placement and by itself is not enough. The publisher's reference to Workplace 10B is not a good fit because that lesson is about dividing a whole into halves.

There is an occasional reference to division but it pops up without a lot of direct instruction. It is briefly mentioned but never developed. Volume 3, Unit 7 does get to dividing candy bars between and among students but the focus is on fractions.

Since ST does not have access to OSPI rationale, it is not possible to know whether reviewers looked at the same material and came to different conclusions or did not see the same material.

Standard 2.2.C illustrates the point of people looking at the same material and drawing different conclusions. While ST reviewers saw a very weak content match—0.17—between *Bridges* and the standards, OSPI reviewers saw a strong—0.75—content match. The rationale for OSPI's score isn't

available, but we do know that reasonable people can look at the same evidence and draw different conclusions, since it appears that is what OSPI and ST reviewers did.

In some instances it is also possible that ST looked at more material than did OSPI. Both groups looked at the materials in the publisher's alignment. Additionally, ST reviewed every lesson, including those in the supplementary materials, which was not possible for OSPI reviewers. Also, in the particular case of *Bridges*, materials that should have been excluded, because they were replaced by supplemental units, might inadvertently have been included by OSPI reviewers.

To understand better what happens when ST identifies more content than OSPI, disparate scores for grade four *Expressions* are examined more carefully. This program and grade level were chosen because that is where the largest discrepancy lies.

ST judged *Expressions* to have 88 percent of the content related to multi-digit multiplication versus the 58 percent of content OSPI scored present. While generally the difference for this topic is an accumulation of slightly higher ST scores, there are some glaring exceptions.

4.1.E Compare the values represented by digits in whole numbers using place value.

(OSPI score = 0.000; ST score = 1.000)

ST Comments:

> "Understand the value of the digit" is emphasized when decimals are taught in unit 3. Beginning on page 1047 to 1086 decimals are thoroughly taught. This is a place value based unit, that includes modeling, comparing, impact of zeros, and lots of practice via decimal secret code cards. There is optional extension into thousandths. In addition to inclusion in the spiral review, there is a follow-up lesson on pg. 1139.

> Using place value with numbers is a focus of Unit 3 and lays a strong foundation for comparisons. Understanding the value of the digits and how they compare in later units (Unit 11) as they relate to decimals along with the decimal Secret Code Cards makes this content fully covered.

4.3.E Demonstrate that rectangles with the same area can have different perimeters, and that rectangles with the same perimeter can have different areas.

(OSPI score = 0.125; ST score = 1.000)

ST Comments:

> This is really how the program introduces area, but the non-relationship is explicitly taught on 245 & 246

> This content is explicitly addressed on p. 245-246.

4.4.F Describe and compare the likelihood of events.

(OSPI score = 0.000; ST score = 0.833)

ST Comments:

> Unit 9 focuses on probability and the likelihood of events. There are two pages of practice for students—more is needed as this can be a difficult concept for students.

> Unit 9 is about probability and covers equally likely, probability, sample space, and outcomes with typical activities like penny tosses and drawing marbles. It includes area probability using spinners. It ends with students designing a fair game. Although no single lesson focuses on comparing the likelihood of events, it is embedded and questions are sprinkled throughout. TE 945 to 956

Since the content is clearly present, one has to believe that ST reviewers saw lessons and units that OSPI reviewers did not see.

If it were possible for everyone who reviewed a program to discuss their scores and to share his or her rational, ST believes the scores would be much more similar.

The point of this discussion is to examine instances where ST and OSPI disagree. As is clear, there are discrepancies in many places, but there seems to be feasible explanations for those situations. The two groups agree more often than not and there are logical reasons when discrepencies do occur. It is not necessary to have perfectly coordinated scores for ST to be able to corroborate OSPI's work.

Elementary School Programs
Mathematical Soundness

The majority of the attention during OSPI's curriculum review and ST's subsequent study of curriculum review is focused on matching the program content to the standards.

The mathematical analysis is an additional review of the material that looks carefully at selected topics across grade levels. It goes beyond the simple content match to determine whether central topics are correct from a mathematical viewpoint.

The analysis is not about how the mathematics is taught—a question better answered by mathematics educators—but about how accurately mathematics is represented in the program. A mathematician is best prepared to make judgments about the mathematical soundness of representations of concepts and the accuracy of definitions. He or she notices when critical explanations are missing.

ST selected three topics at the elementary school and two at middle school to be thoroughly analyzed from their introduction through closure of the topic. Grade placement, while noted, was not of particular import.

For elementary school, the topics of whole number multiplication, area of the triangle, and the arithmetic of fractions were selected. A detailed discussion of each is found in Appendix A.

The following chart offers a visual overview of the results of ST's examination. It is based on the Mathematical Soundness Report available for download at the website of SBE and ST. Please note it does not reflect or relate to a summary of scores—no rubric was used for this component of ST'S work. Detailed analysis of each of the programs can be found beginning on page 31.

	Whole Number Multiplication	Area of Triangle	Adding and Subtracting Fractions
Bridges In Mathematics	+	–	–
Investigations	√	√	√
Math Connects	+	–	+
Math Expressions	+	+	+

+ All necessary concepts and skills related to the topic are introduced, developed, represented, and consolidated using correct mathematics.

√ The topic is introduced and consolidated without mathematical errors, but some aspect is not thoroughly developed.

– The typical student would not master the content or be ready for the next level of mathematics.

Math Expressions

Of the four programs, *Math Expressions* is mathematically the strongest. With great clarity and simplicity the program goes from a geometric representation of multiplication using place value through a sequence of notations to end up with the standard algorithm for whole number multiplication.

The thread related to areas of triangles is completely covered. The formulas for the areas of triangles, parallelograms, and rectangles are each thoroughly developed. The area of the triangle is related to that of the parallelogram and this to the area of a rectangle. All of this is done with clarity.

The content in Math Expressions is not just present and correct, it is elegantly done.

The program also does an excellent job of developing the addition and subtraction of fractions. Fractions are defined as numbers by using fraction strips and the number line. Addition and subtraction are represented on the number line as well. There are good problems.

With *Math Expressions,* students should be prepared for the next levels of mathematics.

Math Connects

Math Connects covers the thread related to multiplication with a minor exception. It lacks a numerical model for multiplication that contains place value and demonstrates commutativity and distributivity all in one place.

The definition and the formula for the area of a rectangle are well done, but the formula for the area of a parallelogram is not given. Students are shown how to compute the area, but the formula is not made explicit. The real failure of *Math Connects* is the lack of content about the area of a triangle. This thread is mathematically sound, but incomplete.

Fractions and the arithmetic of fractions are thoroughly covered. A minor weakness is the emphasis on a least common denominator for adding and subtracting fractions. The easily accessible common denominator is covered, but its value is downplayed.

In all, with the exception of areas of triangles, students should be prepared for further mathematics when coming from *Math Connects*.

Bridges in Mathematics

Bridges in Mathematics covers most of the whole number multiplication thread in Grade five of the original program and finished it in the *Bridges Washington State Supplement*. The transitions from representations to partial products to the standard algorithm are handled nicely. *Bridges* lacks a numerical model that demonstrates the use of commutativity and distributivity and work with three-digit numbers.

It should be mentioned that the program does not celebrate the standard algorithm, but seems to always be looking for alternative ways to do things. Giving students the opportunity to develop and learn a variety of procedures does not require devaluing the standard algorithm.

The representation of fractions and their addition and subtraction, through representations, is nicely done in Investigations with their clock model and their rectangular grid model.

In its *Washington Supplement*, *Bridges* properly defines area and derives the formula for the area of a rectangle. Unfortunately, the derivation of the formula for parallelograms is mathematically problematic and the derivation of the formula for the area of a triangle is inadequate because it is based on the very limited examples of Geoboard triangles. There are no word problems.

Bridges' grade-three supplement explains well what it means to put fractions on the number line, but falls short with representing adding and subtracting fractions as numbers. There is inadequate opportunity for students to understand addition and subtraction, partly because the concepts and skills associated with common denominators are not developed and partly because there are no word problems for fractions or area after the concepts are developed.

In its Washington Supplement, Bridges properly defines area and derives the formula for the area of a rectangle.

In the grade five *Washington Supplement* materials reviewed by ST, there is a mathematical error in the primary explanatory example of fraction addition. ST understands this error has now been corrected and applauds the publisher's responsiveness.

While Bridges provides the necessary foundational skills and concepts for whole-number multiplication it does not do so for fractions or area of a triangle. The materials need more supplementation before ST believes the typical student would meet Washington's standards in these two threads.

Investigations

The strength of *Investigations* in the multiplication thread is the development of strategies for single-digit multiplication leading to fluency. The primary weakness is that the program does not lead to fluency with the standard algorithm. Although a supplementary activity does a nice introduction to the standard algorithm, relating it to the partial products algorithm and the place value area representation for multiplication, it is a standalone activity that is not mathematically incorporated into the program. The main program continues to develop multiple strategies, unaware that the standard algorithm has been developed, without providing the concentration necessary to provide fluency.

With the help of a one-page supplement, the formulas for the areas of rectangles, parallelograms, and triangles are developed. The work with triangles is weak: the height of a triangle is not defined. There are an inadequate number of good problems.

The representation of fractions and their addition and subtraction, through representations, is nicely done in Investigations with their clock model and their rectangular grid model. However, this follows weak and confusing work in grades three and four that fails to give students a solid start. Work in *Investigations* is limited to fractions with sums of less than 2 and there is very limited work with mixed numbers, leaving students unprepared to deal with fractions as simple as 11/3. Most importantly, common denominators are not well-developed, leaving students ill-prepared to add arbitrary fractions with what they are taught. Students using this program will be not be well prepared to go on in mathematics.

Middle School Findings
Comparison of Content Alignment

The following chart provides a comparison of the OSPI and ST results related to the amount of content match identified by the reviewers.

Grade 7	OSPI Scores by Section	ST Scores by Section	Difference
Holt			
7.1 Rational numbers and linear equations	0.93	0.79	-14%
7.2 Proportionality and similarity	0.57	0.78	21%
7.3 Surface area and volume	0.94	1.00	6%
7.4 Probability and data	0.94	1.00	6%
7.5 Additional key content	0.94	1.00	6%
7.6 Reasoning, problem solving, and communication	0.95	0.94	-2%
Holt Grade 7 Average Scale Score	**0.85**	**0.89**	**4%**
Math Connects			
7.1 Rational numbers and linear equations	0.71	0.74	2%
7.2 Proportionality and similarity	0.64	0.80	16%
7.3 Surface area and volume	0.59	0.50	-9%
7.4 Probability and data	0.93	0.77	-16%
7.5 Additional key content	0.88	1.00	13%
7.6 Reasoning, problem solving, and communication	0.86	0.60	-26%
Math Connects Grade 7 Average Scale Score	**0.75**	**0.71**	**-4%**
Math Thematics			
7.1 Rational numbers and linear equations	0.81	0.74	-7%
7.2 Proportionality and similarity	0.43	0.74	31%
7.3 Surface area and volume	0.63	0.58	-4%
7.4 Probability and data	0.80	0.87	7%
7.5 Additional key content	0.92	1.00	8%
7.6 Reasoning, problem solving, and communication	0.98	0.81	-17%
Math Thematics 7 Average Scale Score	**0.73**	**0.77**	**4%**
Prentice Hall			
7.1 Rational numbers and linear equations	0.82	0.81	-1%
7.2 Proportionality and similarity	0.63	0.72	10%
7.3 Surface area and volume	0.31	0.58	27%
7.4 Probability and data	0.65	0.93	28%
7.5 Additional key content	0.88	1.00	13%
7.6 Reasoning, problem solving, and communication	0.59	0.79	20%
Prentice Hall Grade 7 Average Scale Score	**0.64**	**0.78**	**14%**

ST and OSPI results were very similar for grade seven. The two groups differed by less than four percent for *Holt*, *Math Connects*, and, *Math Thematics*. While there was more of a difference, OSPI and ST were still relatively close (0.64 vs 0.78, respectively) in their judgments related to the content match in *Prentice Hall*.

At the topic level, there are many more differences. For some topics in some programs OSPI found a stronger content alignment than did ST. For other topics, ST saw a better match. The only consistency ST notes is that ST found a better—it ranges from 10% to 31%—content alignment related to the topic of proportionality and similarity than did OSPI.

In order to investigate more fully whether different reviewers looked at different materials or whether they looked at the same materials and came to different conclusions, ST compared reviewer notes and publisher alignments. ST reviewers were under no compunction to record all supporting lessons, but the details in the scoring sheets suggest they often did so. The following table compares *Holt's* and *Prentice Hall's* alignments and ST citations for the topic of proportionality.

Nothing is certain, but the following table which compares the materials likely seen by each group of reviewers seems to indicate that for the topic of proportions, the higher scores granted by ST relate to both the review of extra material and to different judgments about the same materials.

There are several instances, such as in standard 7.2.C for both *Holt* and *Prentice Hall*, when ST looked at material beyond what is listed in the publisher's alignment and awarded a higher score. Sometimes this material was completely omitted by the publisher, as was the case for *Prentice Hall* and standard 7.2.H, but more often ST found related material in the lessons leading up to and away from those cited by the publisher.

There are also times, such as in standard 7.2.F for *Holt*, that both groups appear to have reviewed the same material and ST reviewers granted higher scores.

	Holt Alignment	ST Citations for Holt	Prentice Hall Alignment	ST Citations for Prentice Hall
7.2.C	• Lab 5.7 pp. 298-299 • Lesson 5.7 pp. 300-307	• 5-7 with lab • 5-8 • Reteach, practice, challenge, and problem solving resources.	• 5.5a • 5.6a • Pg. 270 - 271	• 5-5a • 5-5b • 5-6a • Practice sheets, adapted practice sheets, reteaching sheets, and guided problem solving
	OSPI score = 0.88	ST score = 1.00	OSPI score = 0.50	ST score = 0.83
7.2.D	• Lesson 5.9 pp. 308-311	• 5-9 • Chapter 8 extension	• 5.5b • 5.6a • Pg 270 - 271	• 5-6 • Practice sheets, adapted practice sheets, reteaching sheets, and guided problem solving
	OSPI score = 0.63	ST score = 0.67	OSPI score = 0.75	ST score = 1.00
7.2.E	• Lesson 12.5 pp. 650-654	• 5-6 lab • 4-4 • 4-5 • 4-6 • 5-3	• 9-1 • 10-1 • Problem Solving Application	• 9-1 • 9-1a • 9-2 • 9-3 • 9-4 • 9-5 • 9-6 • 9-7 • 9-8
	OSPI score = 0.50	ST score = 0.67	OSPI score = 0.88	ST score = 0.83
7.2.F	• Lesson 5.3 pp. 278-282	• 5-3	• 9-1 • 9-4a	• 10-2 • 10-2b • 10-3 • 10-3a • Practice sheets, adapted practice sheets, reteaching sheets, and guided problem solving
	OSPI score = 0.250	ST score = 0.667	OSPI score = 0.50	ST score = 0.50
7.2.G	• Lesson 5.3 pp. 278-282	• 5-2 • 5-3	• 5-2 • 5-4 • Guided Problem Solving	• 5-2 • 5-2 Extension • 5-3 • 5-3b • 5-4a • 5-4b • 5-5
	OSPI score = 0.50	ST score = 0.667	OSPI score = 0.38	ST score = 0.33
7.2.H	• Lesson 5.4 pp. 283-286 • Lesson 12.5 pp. 650-654	• 5-4 • 5-7	• 9-2 • 10-2	• 5-1 • 5-2 • 5-3 • 5-3b • 5-4a • 5-5 • 5-5b
	OSPI score = 0.750	ST score = 1.00	OSPI score = 0.50	ST score = 1.00

Middle School Findings
Mathematical Soundness

	Multiplication and Division of Fractions	Proportions
Holt	√	√
Math Connects	—	+
Math Thematics	—	—
Prentice Hall	√	√

+ All necessary concepts and skills related to the topic are introduced, developed, represented, and consolidated using correct mathematics.

√ The topic is introduced and consolidated without mathematical errors, but some aspect is not thoroughly developed.

- The typical student would not master the content or be ready for the next level of mathematics.

Overall, *Math Connects* is strongest, with *Prentice Hall* and *Holt* tying for the middle place.

It is disappointing to see the treatment of multiplication and division of fractions across the four texts chosen for examination. While all four programs make a good start on the topic, in the end, the rules for the multiplication and division of fractions are just stated and not explained. Students need the opportunity to learn to understand the procedures they are asked to apply. As they stand none of these programs completely does this, although *Holt* and *Prentice Hall* come closest by solving equations of the form BX = A where B is a fraction. None of the programs use the inverse nature of multiplication and division. The good news is that this topic is relatively easy to address with supplementation.

Ratios, rates, proportions and proportionality reach into all corners of middle school mathematics. Helping students see and use the connections among scale and slope and similarity is a complex task. It cannot be amended with a few supplemental lessons and added definitions. This is a topic that must be well developed within the chosen mathematics program.

Holt

In *Holt*, the multiplication of simple fractions is nicely modeled, but the text gives no explanation for the general rule for multiplying fractions. The only attempt to explain the division of fractions is for the cases where the answer is a whole number. The rule for dividing fractions is given without explanation, as a rule to be memorized. One section on solving simple equations could be turned into an explanation of division. It contains

equations of the sort A = BX that are solved by multiplying both sides by the reciprocal of B. This could easily be tied to the inverse nature of multiplication and division, but it is not taken advantage of in this way. The section could also be extended by replacing A, which is usually a whole number, with a fraction.

Holt does not properly define "rates" when they are first introduced. There are a lot of proportion problems, but they are of mixed quality and some are mathematically inappropriate. Cross products are taught, but the underlying logic is not included in the student text. Linear equations, their graphs, and slopes are included, but underdeveloped.

Proportions, slopes, and graphs are also included but the program does not establish well enough the connections that should be made through similar triangles and the proportionality constant. Direct variation is included.

Overall, the content is present, but the concepts and the connections among the concepts need further development to really prepare a student to go on in mathematics.

Math Connects

Multiplication of simple fractions is nicely modeled in *Math Connects*, but there is no explanation for the general rule for multiplying fractions. The only attempt to explain the division of fractions is for the cases where the answer is a whole number. It does not include problems like A = BX, which are solved by multiplying both sides by the reciprocal of B, and at least implicitly make the connection to the inverse relationship of multiplication and division. There is no explanation given for the rule for dividing fractions, when it could, again, be done with an explicit connection to the inverse operations of multiplication and division.

Math Connects thoroughly develops ratios, rates, and proportion, including proper definitions in the text each year of the middle school program. Proportion problems are numerous, of varying types, and such problems are present in all three years of the material. Cross products are appropriately and correctly explained. The connection to a proportionality constant and a graph is not emphasized, but it is there and the connection between slope and similar triangles is also present, although it is even weaker.

Math Thematics

Math Thematics includes solid modeling of multiplication of simple fractions, but again, without explaining the general rule for multiplying fractions. Similar to other programs that ST reviewed, the only attempt to explain the division of fractions is with examples in which the answer is a whole number. The text gives no explanation for the rule for dividing fractions.

Math Thematics 1 sets up a confusing foundation for ratios, rates and proportions by not giving a definition of "ratio". This is corrected in Book 2, a year later, but that delay puts students in the position of having built misconceptions. Cross products are taught, but how and why they work is not explained. There are proportion problems in limited number throughout the three years, but many of them are scattered throughout the text, meaning there is never any intensive practice.

There is a minimal connection of graphs to ratios and rates, but there is nothing explicit about how they connect to proportions, such as the constant of proportionality. Linear equations are not shown to produce graphs that are lines, and no reason is given for slopes of lines to be independent of points chosen to compute them. This series' presentation of proportions would not develop the foundational understanding students need.

Prentice Hall

Prentice Hall includes good models of multiplication of simple fractions, but it does not provide an explanation for the general rule for multiplying fractions. Similar to the other reviewed programs, the only attempt to explain the division of fractions is by using examples that have a whole number for an answer. No explanation is given for the rule for dividing fractions. There is a missed opportunity in a section on solving simple equations that could have easily been turned into an explanation of division. This is a place that the skilled teacher can add to the explanation and make the connections that are missing. Information in the teacher's edition indicates an awareness that these procedures can be explained, but the explanations are not included. The teacher's edition does make the statement that multiplication and division are inverse operations, but this information is not reflected in the lessons.

The definitions for ratios, rates, and proportions are included and correct. Cross products are not explained in grade six, when they are first taught, but they are in subsequent grade levels. There are a variety of word problems, although they are not particularly challenging. The slope is not well developed and defined, nor are equations shown to give lines as graphs, but the real weakness in *Prentice Hall* comes from the lack of connection between proportions and graphs.

Individual Mathematics Programs

Bridges in Mathematics (K-5)

With the help of the supplemental materials, *Bridges* generally covers the thread of whole number multiplication. This is not true for the topic of area. The supplemental materials include the formulas for area missing from the main program, but the derivation of the formula for the area of a parallelogram is mathematically problematic and the derivation of the formula for the area of a triangle is inadequate. The supplemental material address the addition and subtraction of fractions, but common denominators are not developed. There are three sample problems that each illustrate a central point in a different type of fraction problem. One of these contained an error, demonstrating the weight each example bears when a program has a limited number of examples. The typical student would only be prepared for the thread of whole number multiplication and would need more support for the other two topics.

OSPI Grade 2 Average Scale Score	0.86
ST Grade 2 Average Scale Score	0.65
OSPI Grade 4 Average Scale Score	0.62
ST Grade 4 Average Scale Score	0.88

Bridges is a program structured around eight units each year. Units are organized by mathematical strands like fractions and data, rather than themes or problem contexts. Typical lessons begin with a problem or investigation that small groups of students tackle at their desks. While students share their thinking, strategies, and solutions in the classroom discussion that follows, there does not appear to be a systematic emphasis on bringing closure to or consolidating the mathematical ideas being studied.

Direct instruction is rarely used in the main *Bridges* program and occasionally used in the supplemental *Bridges* materials. Most days include time for students to work at centers that have games and activities related to the mathematical content under investigation in order to practice skills and explore ideas.

Bridges offers its users a lot of information, beginning with its program overview. The teacher's edition contains a well-developed unit overviews that precede each unit and that teachers can draw upon daily. There is information about lesson-specific classroom management and assessment tips embedded throughout. Most lessons include examples of the types of classrooms discussions that should be generated during class. Because the classroom discussions are a mainstay of the program, these are important examples.

ST was most impressed by the publisher's creating lessons that exactly match to Washington's standards.

On the negative side, ST reviewers found the materials hard to navigate. The program exists in multiple books—1) number corner activities, 2) main materials, 3) practice workbook, 4) home connections, and 5) the supplemental materials— and lacks both a comprehensive table of contents and an index. It would be difficult for a teacher to isolate topics for review or extension and to remember that some lessons in the main program must be replaced with lessons from the extensive supplemental materials.

Bridges sometimes failed to include explanations of the mathematics in the teacher's edition. Explanations and worked examples were almost always missing from the student materials.

Holt (6-8)

Holt begins by modeling well multiplication of simple fractions, but it does not bridge from the model to the general rule. Division of fractions is only explained for those problems with whole number answers, leaving students to assume that if it works for these numbers it must work for all numbers. Although the paper folding model is included and teachers are reminded that division is multiplication by the reciprocal, there is no explanation for dividing fractions included. There is a section on solving simple equations that could be turned into an explanation of division and in some ways comes close, but *Holt* does not make this connection. Both multiplication and division are broken down into small bits for easier digestion, making it the best of the four programs in this respect.

Rates are not properly defined when they are introduced. There are a lot of proportion problems, of mixed quality. Cross product is not explained in the student text, but it is in the teacher edition. Linear equations, their graphs, and slopes are done, but not done thoroughly. Proportions, slopes, and graphs are not linked up well through similar triangles and the proportionality constant, but the concept of direct variation is included. Students would be prepared to go on, assuming teachers made explicit the implicit content.

OSPI Grade 7 Average Scale Score	0.85
ST Grade 7 Average Scale Score	0.89

Lessons usually begin with a warm up activity that activates prior knowledge and includes a short problem of the day. Using direct instruction, the teacher demonstrates and guides students through a set of about three example problems, each one related to a different sub-topic, that illustrate how to solve a problem type. For example, the lesson on customary measurements has a sample problem for 1) choosing the appropriate customary unit, 2) converting customary units, and 3) adding or subtracting mixed units of measure. This is followed by guided practice, independent practice and then practice problems and spiral review.

There is a dependence on direct instruction with guided and independent practice, although the program includes some games to solidify skills. Units are by topic and each lesson stands alone. It usually builds on the lesson from the previous day, but sometimes the effect can be a bit jumbled. Chapter five includes lessons about ratios, rates, slope and rates of change, identifying and writing proportions, solving proportions, generating formulas to convert units, customary measurements, similar figures and proportions, scale drawings and scale models, and proportional relationships. The connections among and between these topics are not as clear as they should be.

This program appears easy to implement. The teacher's edition is rich with calls-outs related to math background, interdisciplinary connections, ongoing assessment, student intervention, and teaching tips. There is a well-developed section on test-prep including various types of problems and tools to aid teachers such as student work samples for extended response items.

A fair amount of content is included in *Holt* that does not map to Washington standards. This includes a lot of chapter five, which is about angles, circles and polygons, congruent figures, translations, reflections, rotations, and symmetry; most of chapter six, which is about percent, percent of change, and interest; and most of chapter nine, which is about measurement of two-dimensional figures. However, the program is designed in such a way that it is relatively easy for a teacher to skip chapters, as long as they know to do so, meaning the content does not have to take time away from teaching Washington's standards. The student texts do not always contain enough explanations of the mathematics and worked example problems to enable the determined student to teach him or herself.

Investigations (K-5)

Investigations receives the lowest scaled score given for content from both OSPI and ST of the four top-ranked middle school programs. Fluency with the standard algorithm is not developed, although all of the necessary components of whole number multiiplication are there. *Investigations* handles the topic of the standard algorithm in a one-page supplementation in grade four and one class period in grade five. Foundational concepts for area are well-developed in the main program. Supplemental activities provide practice with formulas for the areas of rectangles, but fail to properly develop the formulas for areas of parallelograms and triangles. In particular, the height of the triangle is not defined. Addition and subtraction of fractions has a good start with strong models, but this foundation is not brought to closure with good development of common denominators. A typical student would not be well prepared for the next level of Washington mathematics using this program.

OSPI Grade 2 Average Scale Score	0.61
ST Grade 2 Average Scale Score	0.63
OSPI Grade 4 Average Scale Score	0.50
ST Grade 4 Average Scale Score	0.51

A typical day begins with "10 Minute Math" which gives students the opportunity to maintain and practice learned skills and numerical fluency. Most of the remainder of the day is spent with students working in groups to investigate a mathematical situation or problem, which is different than applying mathematics they know to solve word problems or a problem based on a realistic context.

Investigations, as the name suggests favors an approach to learning that is exploratory in nature. There is an emphasis on students discussing their strategies and solutions and skilled, knowledgeable teachers are critical to guide discussions and consolidate student understanding. Explanations of the mathematics and worked sample problems are rarely included in the student materials.

The strength of the program is in the suggestions for conceptual development of key concepts. It is hard to stress how well this is done in *Investigations*. Students develop strong visual models for challenging concepts such as fractions, multi-digit multiplication, and area. However, the transition to abstract fluency or generalization is not always developed or evident. For example, students develop good number sense, but may not develop efficiency in operations.

Two other issues should be mentioned. Pacing may prove challenging. While each lesson is appropriately designed for a class period, the teacher is frequently asked to identify additional problems and to provide enough time for struggling students to understand the concept. At the same time, there are insufficient extensions or enrichments to engage adept students who may have already learned the concepts.

Also, at some grade levels *Investigations* includes a great deal of content that does not align to Washington's standards. For example, as much as half of the content in grade four Investigations matches to standards at other, usually lower, grade levels, suggesting *Investigations* teaches content later in a student's career than the state's standards require.

Math Connects (K-8)

Math Connects is the only program ranked in the top four for both the elementary and middle school levels.

At the elementary level, the thread related to multiplication is covered except it lacks a numerical model for multiplication that contains place value and demonstrates commutativity and distributivity all in one place. The definition and the formula for the area of a rectangle are well done, but it doesn't build to finding the area of a triangle. Students are shown how to compute the area of a parallelogram, but the formula is not made explicit. How to find the area of a triangle is not taught, making this thread mathematically sound but incomplete. Fractions and the arithmetic of fractions are thoroughly covered. One minor flaw is that *Math Connects* emphasizes the least common denominator for adding and subtracting fractions at the expense of the easily accessible common denominator.

In middle school, *Math Connects* makes a good start with excellent modeling of multiplication of simple fractions but it lacks an explanation for how this extends to the general rule for multiplying fractions. Ratios, rates, and proportion are properly defined every year and proportion problems are numerous, of varying types, and in all three years of the middle school program. Other topics related to proportionality need slightly more explanation or development. Cross products are used and explained properly. The connection to a proportionality constant and a graph is made, but not emphasized enough and the connection to slope and similar triangles is there with even less development.

*Math Connec*t needs a touch of supplementation to be fully mathematically developed, but the areas it is missing are easy to supplement.

OSPI Grade 2 Average Scale Score	0.76
ST Grade 2 Average Scale Score	0.78
OSPI Grade 4 Average Scale Score	0.76
ST Grade 4 Average Scale Score	0.76
OSPI Grade 7 Average Scale Score	0.75
ST Grade 7 Average Scale Score	0.71

With *Math Connects*, lessons have a similar structure in the elementary and middle schools. On a typical day, the teacher begins with some scaffolding questions that active prior learning. Next the teacher works some examples, asks the class a few questions to check for understanding, students work some problems to be sure they know what to do, and then the students work practice problems which include some spiral review problems. While every program has a predictability to its structure, this one seems to suffer from adherence to its format.

This program depends on direct instruction, even at the elementary level. Hands-on activities are included in the teacher's edition but identified as appropriate for use with below grade students. At the middle school level, students participate in approximately six mini-labs per year, designed to last about one-half hour, scheduled before a new major concept is introduced. With the exception of the mini-labs, students spend much of their time on the procedural side of mathematics.

The program appears relatively easy to use, although the teacher's edition has so much information—not all of it helpful—that a teacher might be overwhelmed. The directions to the teacher are often very general, and often not specific to the problem or concept. While there is a good deal of information about differentiation—some useful, some not - there is not enough good information about the whole group instruction. The additional worksheets provide plenty of additional material for teachers to use to reteach and enrich.

Math Expressions (K-5)

Math Expressions develops whole number multiplication completely and elegantly. The area thread is done extremely well. Adding and subtracting fractions, both the conceptual and procedural sides as well as the connections between the two, is explicitly developed. With the exception of the work on common denominators, all of the mathematics is in the student materials. This program provides a solid foundation to move ahead in mathematics.

OSPI Grade 2 Average Scale Score	0.69
ST Grade 2 Average Scale Score	0.80
OSPI Grade 4 Average Scale Score	0.53
ST Grade 4 Average Scale Score	0.90

A typical day starts with a short skills building or practice and homework review. The actual lesson usually starts with a problem that students work on, in groups, at their desk. This is followed by "math talk," *Expressions'* version of student discourse, during which the teacher pulls out student thinking and summarizes and consolidates the important mathematical ideas for the day. Teachers sometimes use direct instruction during this time to teach specific skills or concepts. Students are then led through some guided practice or group work and on to independent practice or a group task. Every lesson has a homework worksheet and a spiral review worksheet.

The teacher's edition is very strong. Although the front material is a bit overwhelming, there is a complete table of contents and index, so content is very easy to find. The goal for each lesson is clearly identified, as is a suggested time frame and a list of specific materials that will be needed. "Best practice" ideas, such as the main points that should be included in the class discussion, are woven into the teacher's edition. Sometimes it alerts the teacher to commonly held student misconceptions and ideas to support the student. Sometimes it offers general guidance on how to teach a lesson like, "Students will also probably say that both arrays have six dots; accept this answer because it works for these specific arrays. But it cannot be the basis for a general argument for all arrays asked for in the next exercise."

Math Thematics (6-8)

Math Thematics does a good job of modeling multiplication of simple fractions and giving students the opportunity to practice doing so in a meaningful context. What is missing is the bridge between the model for the simple case and the explanation for the general rule. The only attempt to explain the division of fractions is for the cases where the answer is a whole number. No explanation is given for the rule for dividing fractions.

Book 1 sets up a confusing foundation for ratios, rates and proportions by not giving a definition of ratio. This is corrected in Book 2, but by then students are working from a shaky foundation. Why and how cross products work is not explained. There are proportion word problems, but they are limited in number and scattered throughout the material.

A minimal connection of graphs to ratios and rates is made, but nothing explicit is done about proportions, such as the constant of proportionality. Linear equations are not shown to produce graphs that are lines and there is no explanation given for slopes of lines to be independent of points chosen to compute them.

A student would be unprepared to go on in mathematics, although multiplication and division of fractions could be easily supplemented.

OSPI Grade 7 Average Scale Score	0.73
ST Grade 7 Average Scale Score	0.77

Math Thematics takes traditional content and organizes it into themes or modules. For example, the first module, "Search and rescue" uses an excerpt from a book, Hachet, in which a young man is stranded after his plane crashes. The unit relates the math in the unit—including angles and rays, coordinate grids, order of operations, adding and subtracting integers, modeling functions, writing equations to model situations, and using inverse operations to solve one-step equations—to search and rescue situations. In the second module, "Bright ideas," the text bundles many important mathematical discoveries along with the key information about the person

who developed the mathematics, thereby at least exposing students to some math history. The remaining modules are: Codes, The Art of Motion, Recreation, Flights of Fancy, Math-Thematical Mix, and Heart of the City. Every section includes "Reflecting" and "Spiral Review." Reflecting has questions and problems that push and check student understanding. Each unit ends with a series of application problems that requires students to apply the mathematics learned in the unit in a realistic context. For example, the first problem in unit one requires the student to develop a rescue plan with limited daylight and gas for the rescue plane. In this way concepts and skills are put into realistic contexts.

The lessons are highly engaging, but the mathematics gets a bit chopped up and the later units seem fragmented. Also, the organization means that it is difficult to omit the mathematics that is not included in Washington's standards and there is quite a lot of that.

The program is easy to use, with a table of contents and index. There is a lot of help for teachers. The margin in the TE include "Tips from Teachers," "Developing math concepts," which are ideas for how to introduce and teach a topic; "Teaching notes," which are mathematical content background notes for teachers; "common errors," (example: "when substituting a value for a variable in a multiplication expression, students will sometimes forget to perform the multiplication and simply write the number....Remind students to write the multiplication symbol or to use parentheses." which describe not only errors students commonly make, but also ways to address them; "About Key Concepts" gives more information about the mathematics; "Absent Students," which identifies materials to use that compress the missed lesson(s); "Classroom Management" has teaching tips that are germane to the specific lesson; "Differentiated Instruction" proves suggestions for that topic, and "Closure question," which ensures students leave the lesson with the main idea. Unfortunately, not all of the important mathematics is included in the teacher help and not all of it is explained when it is included.

Prentice Hall (6-8)

Multiplication of simple fractions is nicely modeled but no explanation for the general rule for multiplying fractions is given meaning students would not understand why the rules work. Division of fractions is explained only for the cases where the answer is a whole number. No explanation is given for the rule for dividing fractions. There is a section on solving simple equations that comes very close and could easily be turned into an explanation of division. There is an indication, in the teacher's edition, that these formulas can be explained, even though the explanations are missing. Similarly, the teacher's edition makes the statement that multiplication and division are inverse operations, but this information is not included in the student text nor used in the lessons.

Correct definitions are included for ratios, proportions, and rates when a concept is introduced. Cross products are not explained the first year they are taught, but are in the year two. There are a variety of problems but none are particularly difficult. It is left to the teacher to fill in the connections between proportions, slopes, and graphs. Generally, slope is not well defined and equations are not explicitly shown to give lines as graphs. While the material is present, the teacher must do the teaching.

OSPI Grade 7 Average Scale Score	0.64
ST Grade 7 Average Scale Score	0.78

Prentice Hall is a solid mathematics text that contains appropriate mathematics. Activity labs are used prior to the majority of lessons to develop conceptual understanding of mathematical procedures. These are an essential component of the text; it is during these labs that the conceptual base for students' understanding of specific mathematical procedures occurs. Good models exist in the labs; however the "bridge" between the models and the rules are not made explicit in the teaching notes or in the actual lesson. Since fluency may not be certain without conceptual understanding, teachers will need to know to do this.

The actual lessons begin with a summary of "what" will be learned and "why" it is needed, a list of essential skills and vocabulary, and multiple

examples where key concepts are highlighted. In addition to labs, the program uses a mix of direct instruction and student group work that leads to guided and independent practice. Homework exercises offer "Go for Help" and "Go Online" notes to assist students with finding appropriate examples and tutoring help.

The TE includes a "Mathematics Teacher Handbook" at the beginning of the text that explains how to use the teacher text effectively. This section provides information on where to find real-world applications, activity labs, chapter projects, and specific problem solving strategies throughout the text. Support is provided for lesson differentiation, formative and summative assessment, technology use, and additional professional development opportunities. A "Math Background" summary precedes each chapter. This includes essential math understandings for successfully teaching the concepts in the chapter; teachers that by-pass reading these two-page summaries risk omitting important connections between procedures and the conceptual understanding of those procedures. Included in the margins of each lesson are additional math backgrounds relevant to the individual lesson, guided instruction notes to deepen student understanding, differentiated instruction suggestions, lesson openers and closers, common error alerts, additional examples, and assessment tools.

Even with all of this, the teacher's edition often does not make explicit why procedures work nor the connections between various strands of mathematics. The student edition does not stand alone. Among other things, it does not explain what a student should learn in the labs and how that information would be applied in specific lessons. For example, in 2-4a Activity Lab Comparing Fractions, students model comparing fractions with paper strips. The lab includes a discussion of why it is easier to compare fractions with common denominators. In the lesson 2-4, students compare fractions by writing the fractions with common denominators and comparing the numerators. No reference to the lab is made and no alternate strategies are included; the teacher is needed to make the connection.

Appendix A

Whole Number Multiplication

The standard algorithm for multiplication has particular importance in mathematics for several reasons. First, it is fairly easy to link it to the all-important place value system. Second, it is necessary in order to extend multiplication to decimals; the same algorithm is used, it is just the decimal place that has to be understood. Third, whole number multiplication is used for all four arithmetic operations with fractions:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}, \quad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \text{ and } \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}.$$

Fourth, multiplication is used throughout all of mathematics.

The standard algorithm solves the universal problem of whole number multiplication, taking the *ad hoc* out of multiplication. As such, it is one of the truly beautiful and powerful mathematical theorems that students can learn about in elementary school. Learning the algorithm is much more than just learning a way to multiply, it is learning a major mathematical structure.

The thread we consider starts with the following:

3.2.D *Apply and explain strategies to compute multiplication facts to 10x10 and the related division facts.*

This is an important preliminary, as is indicated in the Washington standards, to the following:

4.1.A *Quickly recall multiplication facts through 10x10 and the related division facts.*

The standard algorithm is based on distributivity, commutativity, and these single digit multiplications. Knowing all without hesitation is important for fluency with the standard algorithm. Once these are all under control it is possible to connect multiplication with place value, as is expected in this standard:

4.1.C *Represent multiplication of a two-digit number by a two-digit number with place value models.*

The usual representation is to break up numbers using expanded notation and to use the area model of multiplication, thus connecting multiplication to its foundation in the place value system. There are certainly other acceptable models, but it is essential these include a numerical model to make the step from a representation to numbers.

In the numerical model we need this standard:

4.1.D *Multiply by 10, 100, and 1,000.*

Then, multiplication, as it is represented using place value, must be connected to the notation for the standard algorithm so that we can achieve the standard below:

4.1.F *Fluently and accurately multiply up to a three-digit number by one- and two-digit numbers using the standard algorithm.*

There is a lot that is important here. The notation for the standard algorithm must be connected to a place value representation so that the notation makes sense to students. Obviously, learning this standard by rote is inadequate. This standard will be emphasized as the goal, because without it, even with the foundation of good developmental standards, a student is unprepared to go on in the study of mathematics. Once fluent, multiplication should cease to be a problem but should just be a skill that can be used in problem solving. Thus, ultimately, we need the standard below:

4.1.I *Solve single- and multi-step word problems involving multi-digit multiplication and verify the solutions.*

The point of this thread is to develop an understanding of and a facility with the standard algorithm for multiplication so that it can be readily used as a tool for problem solving. This is of fundamental mathematical importance, and this is reflected by its place in the Washington State standards.

Area of a Triangle

Area is a fundamental concept in mathematics. The area of a triangle is derived from that for a parallelogram (by taking two copies of the triangle and making a parallelogram) and the area for that is, in turn, derived from the area of a rectangle (by cutting up the parallelogram and rearranging the pieces to make a rectangle). The area of a rectangle follows from multiplication and the area of a square, which essentially gives the definition of area.

With the area of rectangles, parallelograms, and triangles under control, the area of polygons and planer surfaces can be calculated. In practice, measurement also comes into play. The computation of volumes frequently depends on knowing the areas connected to the solid. As the student progresses, the techniques for computing area also progress. The Integral Calculus allows students to extend their knowledge of area to much more

complex figures, but that ability depends heavily on the understanding of area from elementary school.

This thread begins with the area part of:

4.3.C *Determine the perimeter and area of a rectangle using formulas, and explain why the formulas work.*

The "explain why the formulas work" part of this is essential. Next, we go to:

5.3.D *Determine the formula for the area of a parallelogram by relating it to the area of a rectangle.*

The goal of this thread is given by the next two standards:

5.3.E *Determine the formula for the area of a triangle by relating it to the area of a parallelogram.*

and

5.3.I *Solve single- and multi-step word problems about the perimeters and areas of quadrilaterals and triangles and verify the solutions.*

Fractions and the Arithmetic of Fractions

The importance of fractions to mathematics cannot be overstated. Students start with whole numbers and then develop fractions to get the rational numbers. Eventually, real numbers can be constructed from the rational numbers and, finally it is possible to add the last bit to get complex numbers. With the real numbers one can put coordinates on the line or the plane, and from this comes the ability turn geometry into algebra and algebra into geometry. The point is that fractions are an essential intermediary step in mathematics.

Numbers and geometry are at the heart of mathematics, and fractions are required for both. You can't do mathematics without an understanding of fractions and their operations. To do fractions requires a thorough understanding of whole numbers and the arithmetic operations with them.

The step of going from whole numbers or integers to fractions is the same step taken to go from polynomials to rational expressions. This is a common transition in mathematics and elementary school fractions are the easiest place to start.

The four arithmetic operations with fractions are divided between grades five and six in the Washington State standards. This crosses the boundary between elementary school and middle school and so it is likely that

different programs will be used for these two grades. That boundary makes it all the more important to accomplish the grade five standards in grade five, and that middle school be able to assume mastery of the grade five standards. Because of this jurisdictional problem, the following is a discussion of the elementary school material.

This thread begins with seeing a fraction as a number, and putting it on the number line will do, as in:

3.3.A *Represent fractions that have denominators of 2, 3, 4, 5, 6, 8, 9, 10, and 12 as parts of a whole, parts of a set, and points on the number line.*

Next fractions need to be defined or represented as numbers in addition and subtraction:

5.2.A *Represent addition and subtraction of fractions and mixed numbers using visual and numerical models, and connect the representation to the related equation.*

Finally we need a purely numerical way to add and subtract fractions:

5.2.E *Fluently and accurately add and subtract fractions, including mixed numbers.*

In order to do this, students must understand what denominators mean and how to find and work with common denominators. We also need the fraction part of the following standard:

5.2.H *Solve single- and multi-step word problems involving addition and subtraction of whole numbers, fractions (including mixed numbers), and decimals, and verify the solutions.*

Multiplication and Division of Fractions

Continuing the fraction thread from elementary school we will focus on multiplication and division for fractions by doing the following:

6.1.D *Fluently and accurately multiply and divide non-negative fractions and explain the inverse relationship between multiplication and division with fractions.*

And by presenting the corresponding fraction part of the application standard

6.1.H *Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.*

In order for word problems with fractions and decimals to make sense for students, there needs to be a good representation of the multiplication of fraction multiplication. It is difficult to represent fraction multiplication for anything but simple fractions so there has to be a logical connection and explanation that takes the student from the simple cases they can see with a representation to the general formula. There are multiple ways to do this, both geometric and algebraic (numerical), but the connection must be established.

Although certain cases of division with fractions can be represented, it is not easy to represent even simple situations such as $\frac{1}{2} \div \frac{2}{3}$.

There are a number of ways to make sense of fraction division but bringing in the inverse relationship between multiplication and division can make it very easy. Any explanation must make sense not only of the simple example just given, but also of examples such as $\frac{2}{9} \div \frac{3}{7}$.

Proportions

After establishing control over the rational numbers in elementary school, middle school includes a lot of work with rates, ratios, percents and proportions. Proportions are reviewed with the focus on student's understanding well enough to work problems, as described in:

7.2.B *Solve single- and multi-step problems involving proportional relationships and verify the solutions.*

To begin, students need to understand what ratios, rates and proportions are and see a good development of representations for proportions. Cross products always show up here and this technique should be justified. Washington would like to see multi-step problems.

Proportions can be put at the center of a number of mathematical connections, as is the case in the standard:

7.2.F *Determine the slope of a line corresponding to the graph of a proportional relationship and relate the slope to similar triangles.*

This means the proportion needs to be represented as linear graphs through the origin. To do this requires an understanding of slope. Slope is shown to be well defined when it is connected to the proportionality of similar triangles. This is required in order to show that a proportionality equation is a straight line. This standard connects much of middle school mathematics.

Appendix B

	OSPI Rubric		ST Rubric
2	**Fully Met** • The standard is fully developed at the expected grade level. A typical student would be able to achieve mastery with the available content.	3	**All content and key teaching/ learning tools present** • The content from the standard is fully present at the correct grade level. • There is adequate information about the content and sufficient teaching/learning ideas included in the program to ensure that students develop conceptual understanding and procedural skill. • There is sufficient practice to ensure mastery. *A typical student would be able to achieve mastery with the core program materials.*
1	**Partially Met** • The standard is fully developed at +/- one grade level from the expected grade. • 50% or more of the standard is met at grade level, but some aspect of the standard is not present. The standard is fully developed, but limited in practice or reinforcement.	2	**All or most content present, but key teaching/learning tools are missing** • The key content from the standard exists in the program at the correct grade level. • The core materials need supplementation to do such things as adding additional opportunities for practice or finding more representations to help students consolidate learning. *Many students would achieve mastery with the core program material.*
		1	**Missing important content** • Some significant aspect of the content is not present. - Some of the content may be completely absent. - Some of the content may be less rigorous. - It would take significant time and knowledge to fill the content gaps in the program. *A typical student would not achieve mastery with the core program materials.*
0	**Not Met** • The standard is not covered at grade level or +/- one grade level. • The standard is briefly mentioned, but a typical student would not be able to achieve mastery with the available content. • The standard is partially met at +/- one grade level.	0	**Little or no content** • All or most of the content in the standard is missing in the program at the correct grade level. - It may be completely absent - It may be briefly mentioned, but it is not developed. - It may contain less sophisticated precursor content that would led to the content in the standard *A typical student would not be able to achieve mastery with the core program materials.*

Appendix C

Average Differences by Grade Level Topics

Grade 2		Grade 4		Grade 7	
2.1. Place value and the base ten system	1%	4.1 Multi-digit Multiplication	11%	7.1 Rational numbers and linear equations	-5%
2.2. Addition and subtraction	-4%	4.2 Fractions, decimals, and mixed numbers	11%	7.2 Proportionality and similarity	20%
2.2. Measurement	4%	4.3 Concept of area	15%	7.3 Surface area and volume	5%
2.4. Additional key content	10%	4.4 Additional key content	16%	7.4 Probability and data	6%
2.5. Reasoning, problem solving, and communication	6%	4.5 Reasoning, problem solving, and communication	16%	7.5 Additional key content	10%
				7.6 Reasoning, problem solving, and communication	6%

This chart shows the average of the differences between OSPI and ST scores by topic. The chart confirms that ST general found a higher degree of alignment than OSPI. Beyond that it does not show other remarkable patterns or inconsistencies.

Appendix D

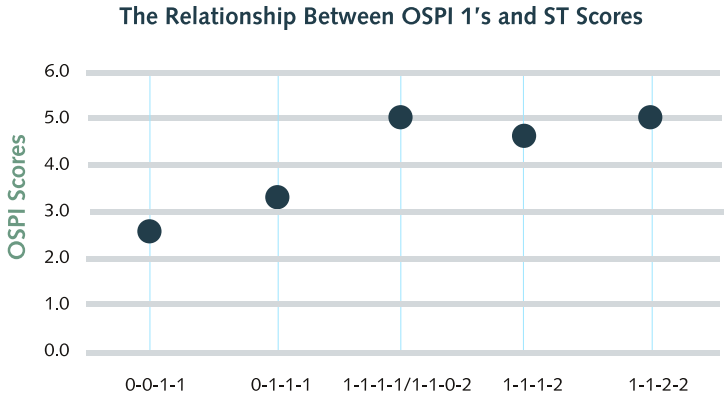
Relationship Between OSPI and ST Scores

ST looked at standards that had been scored "one" by OSPI to see how we scored those standards. To make some sense of this, only those standards to which OSPI gave two "one's" were considered. Put another way, ST looked at all possible combinations of standards scores that include at least two "one's." The score types are:

0-0-1-1 0-1-1-1 1-1-1-1/1-1-0-2 1-1-1-2 1-1-2-2

The possible combinations could have a range of between two and six points given by OSPI reviewers. Scores1-1-1-1 and 1-1-0-2 are considered together because a total of four points is awarded. The ST total scores that mapped to these standards were then averaged. The ST scores ranged from zero, when both reviewers judged there to be no match to six, when both ST reviewers gave the full three points.

The following chart shows a clear relationship. Standards that received a low total score from OSPI tended to receive a low total score from ST. Standards that receive a high total score from OSPI, tended to receive a high total score from ST.



Appendix E

Strategic Teaching Team

Andrew Clark

- *Recently retired K–12 Curriculum Director for Portland Public Schools in Portland, OR*
- *Has taught every grade level using materials from numerous publishers and supervised a variety of programs supported by other programs*
- *Developed and published curriculum materials, but is not associated with any ST reviewed for Washington*

Connie Colton

- *National Board Certified middle school teacher and Department Chair in Omaha, NE*
- *Math Counts coach*
- *Benchmarked and reviewed standards multiple states against national and international documents*
- *Reviewed and benchmarked curriculum for Nebraska and Georgia*

Sean Mulvenon, Ph.D.

- *Professor of Educational Statistics and Research Methods, University of Arkansas*
- *Ph.D. Arizona State University, Measurement, Statistics and Methodological Studies with an interest in the study of longitudinal modeling, structural equation models, and non-parametric statistical procedures*
- *In addition to teaching, directs the National Office for Research on Measurement and Evaluation Systems at the University of Arkansas*
- *Spent the last three years on loan to the U.S. Department of Education, evaluating growth models for state assessment systems.*

Susan Pimentel, J.D.

- *Led strategic planning, reform efforts, and/or standards works in such varied jurisdictions as Ardmore, OK; Allentown, PA; Beaufort, SC; Charlotte-Mecklenburg, NC; Chicago, IL; Elaine, AR; Jackson, WY; Kansas City, MO; Menasha, WI; Red Clay, DE; the District of Columbia, and the states of Arizona, California, Indiana, Ohio, and Maryland*
- *Co-founded Standardsworks*
- *Serves on the Assessment Development Committee of the National Assessment Governing Board*

Linda H. Plattner

- *Reviewed and facilitated standards development in more than twenty states*
- *Reviewed mathematics curriculum in other jurisdictions, including Fulton County, GA*
- *Facilitated school reform in multiple jurisdictions*
- *Taught elementary school before becoming a middle school mathematics teacher*
- *Provided direct support to teachers and schools in California, Ohio and Maryland*
- *Recruited, trained and managed over 100 presenters and their presentation content for the U.S. Department of Eduation's 'Teacher -to-Teacher Summer Workshop and Training Corps'*

W. Stephen Wilson, Ph.D.

- *Professor of Mathematics at Johns Hopkins University*
- *Ph.D. in mathematics from the Massachusetts Institute of Technology*
- *Research area is Algebraic Topology.*
- *In addition to graduate courses and courses for mathematics majors, teaches calculus to engineering and pre-medical students*
- *Completed extensive reviews of standards and analysis of curriculum programs for Fordham and Strategic Teaching*