Edited *Expectations*
For Algebra 1 and Geometry

Washington State Board of Education
May 16, 2008*

*Includes update on next steps based on the State Board of Education Meeting May 14-15

Draft

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The attached *Draft Expectations* are based on the March 5, 2008 version of the *Washington State K–8 Mathematics Standards* developed by the Office of the Superintendent of Public Instruction (OSPI).

The March 5 draft was the work of many people over the past several months. Countless teachers and parents contributed time and effort to produce these standards, which we have praised on many occasions. We recognize how many people have participated in the process and honored their work in our revision.

As stated in our March 10, 2008 report, *Second Review of Washington K-12 Mathematics Standards,* they were good standards that contained important subject matter content. They made it clear that students needed to understand the mathematics they are being asked to do.

The *Second Benchmarking Report*, dated April 11, 2008, confirmed that at the high school level the standards were significantly stronger than the ones Washington had used for the past decade. The March 5 standards scored much higher in rigor, specificity, clarity, depth, grade-to-grade coherence, measurability, and accessibility. The scores related to content did not show the same level of improvement and this has been addressed in the attached revision.

This final revision process, which brings a fresh perspective and involves the Washington Math Panel, will help Washington meet its goal of having standards that are among the best in the world.

During this process we are considering carefully comments from the Washington Math Panel and from the public in general as they offer feedback to the Washington State Board of Education (SBE) via its Web site: www.sbe.wa.gov .

This last revision is intended to:

- Strengthen the content;
- Polish the language;
- Tighten the structure; and
- Remove any vestiges of pedagogy

Structure:
The document is not intended to serve as a course outline. The order of the standards is not intended to suggest an order for instruction. An attempt has been made to order content by increasing sophistication within topics, but the nature of mathematics means it is a messier than one would wish.

To make the underlying structure of the content of Algebra I clear, the standards in this section have been reorganized into new sections that reflect the important topics of the subject. In Algebra I students are introduced to several types of functions, spending considerable time with linear and quadratic functions. In addition to exploring the idea of functions, they learn that specific functions model contextual situations captured in word problems and can be used to solve those word problems. Each new function allows students to solve another class of problems. To highlight this aspect of Algebra I, the standards begin with a section showing the types of problems students learn to solve as they master each type of function.

We offer a second organization of the standards, Version 2, where the types of problems are embedded into the document. While each version has identical content, we prefer the first Version because it better shows the nature of Algebra I.

Content:
Some of the *Expectations* in the March 5 document have been revised, occasionally by combining two or more *Expectations.* A few new standards were written, when it seemed content was missing, and a very few have been deleted. Deleted *Expectations* will be handled in Algebra II, where they can be taught to the depth they deserve. To help the reader, the number of the original *Expectation* is in the lower left-hand corner of the box with the new *Expectation.*

Introductory material:
Strategic Teaching is responsible for one more set of standards: Algebra II. We plan to submit our suggestions for revisions to the introduction to the standards when we submit those standards. In addition to technology, there may be other issues that need clarification and it seems logical to do everything at one time.

Next steps (based on SBE Board meeting May 14-15, 2008):
Strategic Teaching will continue to work on refinements to Algebra I and Geometry and add Algebra II standards. Strategic Teaching will present a full report to the SBE and its Math Panel in June. The Math Panel will meet on June 12th , 2008 to discuss and provide feedback. Strategic Teaching will provide the SBE a final report on the Algebra I, Geometry and Algebra II standards by June 23rd, 2008. The SBE will consider adoption of the final report and hold a public hearing at its July 23-24, 2008 Board meeting. OSPI will use the report to write the new high school standards, which the SBE will consider approving for adoption by July 31st, 2008.

To reiterate the original point: the standards were already good; this revision is intended to provide the final polish to take them to the next level.

Algebra I

A1.1. Core Content: **Solving Problems** (Algebra)

Students learn to solve many new types of problems in Algebra I. They are introduced to several types of functions, spending considerable time with linear and quadratic functions. In addition to exploring the idea of functions, they learn that specific functions model contextual situations captured in word problems and can be used to solve those word problems. Each new function allows students to solve another class of problems. Many problems that appear, on the surface, to be very different from each other can be represented by identical functions? Equations? . This is the beauty and unifying principle of algebra, i.e. that the same algebraic techniques can be applied to a wide variety of different situations.

Deciding what kind of approach best models a particular situation and representing word problems with the corresponding algebraic information is nontrivial mathematical content in itself. The first section highlights the type of mathematical modeling to solve problems that students will be able to do after they learn the concepts and skills in Algebra I.

Expectation	Comments/Examples
A1.1.A Select and justify a function to model a situation, describe appropriate domain restrictions; use the function to solve the problem and interpret the solution in context of the original situation. **(A1.2.F)**	Functions include linear, quadratic, exponential, and those defined piecewise (including functions that contain the absolute value of an expression and step functions). Analyze the rate of change of a function represented by a table or graph to determine whether the function might be linear, and analyze common ratios to determine whether the function might be exponential. Common ratios in an exponential function are determined by calculating the ratio of consecutive entries of the dependent variable in a table corresponding to equal-sized increments of the independent variable. Example: Anderson Lake is a popular lake to catch trout. In 1998, the lake was stocked with 10,000 fish. In the next five years, 15% of the fish were caught by anglers and removed. a. Write a function that models the fish population size at the end of the fishing season each year. Justify the function you wrote. b. State and appropriate domain for the model based on the context.

A1.1.B Solve problems that can be modeled by linear equations or inequalities and interpret the solutions in the context of the problems. **A1.3.G)**	Examples: • The assistant pizza maker makes 6 pizzas an hour. The master pizza maker makes 10 pizzas an hour but starts two hours later than his assistant. They must make 92 pizzas. How many hours from when the assistant starts will it take? **What is a general form of the equation that could be used to determine the number of pizzas for any number of hours (h)?** **Need two or three additional problems to illustrate the type of situations linear functions model.**
A1.1.C Solve problems that can be modeled by a system of two linear equations or inequalities and interpret the solutions in the context of the problems.	• A bird flies 5 miles in 20 minutes into the wind but takes only 10 minutes to fly back with the wind. The bird always flies at a constant rate with respect to the air. How many miles per hour does the bird fly and what is the wind speed in miles per hour?

A1.1.D Solve problems that can be modeled by quadratic equations and interpret the solutions in the context of the problems.
 	
(New)
	Examples:
	• Find the solution where the graphs of y=x+2 and y= x^2 intersect.
	  • Solution: (-1,1) and (2,4).
	• If you throw a ball straight up at 10 feet per second, how long before it comes back to you?
	• Solve for x in $(x-3)^2 = 16$.
	• How long does it take a ball to drop 100 feet?
	• Joe owns a small plot of land 20 feet by 30 feet. He wants to double the area by increasing both the length and the width, but keep them in the same proportion. What will be the new length and width?
	• What 2 consecutive numbers, when multiplied together give the first number plus 16?
	  • Solution: x(x+1)=x+16, so $0 = x^2 - 16 = (x-4)(x+4)$, so there are two solutions: {-4,-3} and {4,5}.

A1.1.E Solve problems that that can be modeled by exponential functions and interpret solutions in the context of the problems. **(new)**	**Students will approximate solutions graphically.** **Need two or three examples**

Draft

A1.2. Core Content: Numbers, expressions, and operations
(Numbers, Operations, Algebra)

Students see the number system extended to the real numbers represented by the number line. They work with integer exponents, scientific notation and radicals. They use variables and expressions to solve problems from both purely mathematical and applied contexts. They extend their understanding of arithmetic operations and properties, enabling them to write and manipulate a wide variety of algebraic expressions. They will demonstrate this understanding throughout high school mathematics as they apply algebraic procedures to solve problems modeled by functions.

Draft

<table>
<tr><th>Expectation</th><th>Comments/Examples</th></tr>
<tr><td>A1.2.A Explain the relationship between real numbers and the number line, compare and order real numbers with and without the number line.

(A1.1.A)</td><td>Students learn that every point on the number line represents a real number, either rational or irrational, and that every real number has its unique point on the number line. They locate, compare, and order the set of real numbers on the number line.

Real numbers include those written in scientific notation or expressed as fractions, decimals, or roots.

Examples:

• Without using a calculator, order the following on the number line:
$\sqrt{82}$, 3π, 8.9, 9, $\frac{37}{4}$, 9.3×10^0</td></tr>
<tr><td></td><td>• A star’s color gives an indication of its temperature and age. The chart shows four types of stars and the lowest temperature of each type.

<table>
<tr><th>Type</th><th>Lowest Temperature (in °F)</th><th>Color</th></tr>
<tr><td>A</td><td>1.35×10^4</td><td>Blue-White</td></tr>
<tr><td>B</td><td>2.08×10^4</td><td>Blue</td></tr>
<tr><td>G</td><td>9.0×10^3</td><td>Yellow</td></tr>
<tr><td>P</td><td>4.5×10^4</td><td>Blue</td></tr>
</table>

List the temperatures in order from lowest to highest.</td></tr>
</table>

A1.2.B Use and interpret letters for different purposes in algebraic expressions and equations. **(A1.1.C)**	Letters can be used: • to represent fixed and temporarily unknown values in equations (such as $3x + 2 = 5$); • to express identities (such as $x + x = 2x$ for all x); • as attributes in formulas (such as $A = lw$); • as parameters in equations (such as the m and b for the family of functions defined by $y = mx + b$); and • as names of universal constants, such as π; • to represent varying quantities (such as x in $f(x) = 5x$) • **to represent functions (such as f in $f(x) = 5x$).**
A1.2.C Evaluate algebraic expressions when variables are assigned specific values and determine all possible values of a variable in a given algebraic expression that assure the expression satisfies a prescribed condition. **(A1.1D and A1.1.F)**	Expressions include those involving polynomials, radicals, absolute values, and integer exponents. Examples: • For what values of a is $\frac{1}{a}$ an integer? • For what values of a is $\sqrt{5 - a}$ **defined?** • For what values of a is $-a$ always positive?
A1.2.D Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to translate among equivalent forms of numeric and algebraic expressions. **(A1.1.G)**	Examples: • $\frac{2^{-2}3^2 5}{2^2 3^{-3} 5^2} = \frac{3^5}{2^4 5}$ • $\frac{a^{-2}b^2 c}{a^2 b^{-3} c^2} = \frac{b^5}{a^4 c}$ • $\sqrt{8} = \sqrt{2*2*2} = 2\sqrt{2}$ • $\sqrt[3]{a \bullet b} = \sqrt[3]{a} \bullet \sqrt[3]{b}$ • **Need an interpretation example**
A1.2.E Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection. **(A1.1.B)**	**Students need to determine whether approximate or exact values of real numbers are appropriate depending on the necessary degree of precision needed in given situations. They also need to decide, after calculating or estimating, if the amount of error is within tolerable limits.**

	Decimal approximations of numbers are sometimes used in applications such as carpentry or engineering, while in mathematics, exact values may be required. Students should understand the difference. For example, 1.414 is an approximation and not an exact solution to the equation $x^2 - 2 = 0$, but $\sqrt{2}$ is an exact solution to this equation. Example: • **Find the length of the diagonal of a rectangular room that is 12 feet by 14 feet. Give your answer to the nearest inch.** • Using a common engineering formula, an engineering student represented the maximum safe load of a bridge to be $1000(99 - 70\sqrt{2})$ tons. He used 1.41 as the approximation for $\sqrt{2}$ in his calculation. When the bridge was built and tested in a computer simulation to verify its maximum weight-bearing load, it collapsed! The student was shocked, as he had estimated that the bridge would hold ten times the weight that was applied to it when it collapsed. Calculate the weight that the student thought the bridge could bear using 1.41 as the estimate for $\sqrt{2}$. Calculate other weight values using estimates of $\sqrt{2}$ that have more decimal places. What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.

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A1.2.F Write algebraic expressions in equivalent forms by using algebraic properties to factor and combine like terms. **(A1.1.H)**	Algebraic properties include the commutative, associative, and distributive properties. Factoring includes: • factoring a monomial from a polynomial: $4x^2 + 6x = 2x(2x + 3)$; • factoring the difference of two squares, such as $36x^2 - 25y^2 = (6x + 5y)(6x - 5y)$ and $x^4 - y^4 = (x + y)(x - y)(x^2 + y^2)$; • factoring perfect square trinomials, such as $x^2 + 6xy + 9y^2 = (x + 3y)^2$; • factoring quadratic trinomials that have a leading coefficient of 1 and factors with integer coefficients, such as $x^2 + 5x + 4 = (x + 4)(x + 1)$; and • factoring trinomials that can be expressed as the product of a constant and a trinomial, such as $2x^2 - 10x - 28 = 2(x^2 - 5x - 14)$ and $0.5x^2 - 2.5x - 7 = 0.5(x^2 - 5x - 14)$.
A1.2.G Write algebraic expressions in equivalent forms by using algebraic properties to perform the four arithmetic operations with polynomials **(A1.1.H)**	Algebraic properties include the commutative, associative, and distributive properties. Students should recognize that expressions are essentially sums, products, differences, or quotients. For example, **the sum $2x^2 + 4x$ can be written as a product, $2x(x + 2)$.** Examples: • $\frac{2x^2}{9} \bullet \frac{6}{2x^4}$ • **More examples are needed that are intermediate in level of difficulty**

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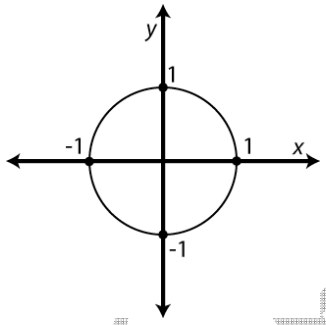
A1.3. Core Content: ***Types and behaviors of functions***
(Algebra)

Students formalize and deepen their understanding of functions, their defining characteristics and uses, and the mathematical language used to describe functions. They learn that functions are often specified by an equation of the form $y = f(x)$, where any allowable x-value yields a unique y-value. Students study linear and quadratic functions. They also learn about absolute value functions, exponential functions, and those that can be defined piecewise. In every case students learn about the representations and basic transformations of these functions and the practical and mathematical limitations that must be considered when working with functions or when using functions to model situations.

Expectation	Comments/Examples

A1.3.A Identify functions, their domains, ranges, roots and independent and dependent variables.
 	
(A1.2.D)
	A function expresses dependence between two quantities, one of which is independent and the other of which is dependent. Functions include linear, quadratic, exponential, and those defined piecewise (including functions that contain the absolute value of an expression and step functions).
	
	Given a problem situation, students should describe further restrictions on the domain of a function that are appropriate for the problem context.
	
	Example:
	• A function $f(n) = 60n$ is used to model the distance in miles traveled by a car traveling 60 miles per hour in n hours. Identify the domain and range of this function. What restrictions on the domain of this function should be considered for the model to correctly reflect the situation?
	• What is the domain of $f(x) = \sqrt{5 - x}$?
	• Which of the following equations, inequalities, or graphs determine y as a function of x?
	  • $y = 2$
	  • $x = 3$
	  • $y = \|x\|$
	  • $x = \|y\|$, where $y > 0$
	  • $x = \|y\|$, where x is a real number
	  • $x^2 + y^2 = 1$

	
	• **Which of the following are functions? Explain why or why not.**
	  • **The age in years of each student in your math class and each student's shoe size**
	  • **The letters associated with the numerals on a telephone**

A1.3.B Represent a function, f(x), as a graph, determine if a graph represents a function. **(A1.2.E)**	Examples: • Describe key characteristics of the graph of $f(x) = \|x - 3\| + 5$.
A1.3.C Represent a function with a symbolic expression, as a graph, in a table, give a verbal description of it, and make connections among these representations. (A1.2.B)	This expectation applies each time a new class (family) of functions is encountered. **Students should be introduced to a variety of functions, including x^3, $\sqrt{x}$, $\frac{1}{x}$ and absolute value functions such as f(x)=|x-3|+5. They should know that f(x) = $\frac{a}{x}$ represents an inverse proportion.** **Translating among these various representations of functions is an important way to demonstrate an conceptual understanding of functions.** Representations include • a symbolic expression that assigns a unique value to each element of a domain, • a table that exhibits a correspondence explicitly for some values of the domain, • a graph in the coordinate plane, and • a verbal description. **Students should know** that a table of values may not uniquely determine a single function without some specification of the nature of that function (e.g., it is quadratic).

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A1.3.D Evaluate *f*(*x*) at *a* (i.e., *f*(*a*)), solve for *x* in the equation *f*(*x*) = *b*, and distinguish between the two.
 	
(A1.2.C)
	Functions may be described and evaluated with symbolic expressions, tables, graphs, or verbal descriptions.
	
	Example:
	• Roses-R-Red sells its roses for $0.75 per stem and charges a $20 delivery fee per order.
	  • What is the cost of having 10 roses delivered?
	  • How many roses can you have delivered for $65?

A1.4. Core Content: ***Linear functions, equations, and inequalities***
(Algebra)

Students understand that linear functions can be used to model situations involving a constant rate of change. They use linear functions to analyze relationships, represent and model problems, and answer questions. They answer these questions by writing and solving equations using algebraic techniques, and extend these techniques to other types of equations. Students also learn graphical and numerical methods for approximating solutions to equations. Students interpret the meaning of problem solutions and explain their limitations. These algebraic skills are applied in other Core Content areas across high school courses.

Revised Expectation	Comments/Examples
A1.4.A Write and solve linear equations and inequalities in one variable. **(A1.6.A)**	This expectation includes the use of absolute values in the equations and inequalities. Examples: • A cup is 6 cm tall, including a 1.1 cm lip. Find a formula that gives the height of a stack of cups in terms of the number of cups in the stack. Find a formula for the number of cups for a given height. Create general forms of your formulas that would work for any cup with overall height *h* and lip height *c*. • Joe and Sue sell grape, cherry and lime snow cones in proportions 2:3:5. They start the day with 80 fluid ounces of syrup. How much of each kind should they take? • Maddie is watching cars go by on the road in front of her house. She notices that one car goes by every second. She knows that the cars are going 30 miles per hour.

	• How many feet apart are the cars? • How many cars are on one mile of roadway? • Write an absolute value equation or inequality for • All the numbers 2 units from 7, and • All the numbers that are more than *b* units from *a*. • Solve $\|x - 6\| \leq 4$ and graph the solution on the number line. • Solve for x when 3x-5=4x+9. • Write an equation or inequality that has • No real solutions • Infinite numbers of real solutions • Exactly one real solution • **Solve for x when 5 < |x+3| < 10.** **(Answer: The intervals 2 < x < 7 and -13 < x < -8)** • **Solve for x in 2(x-3)+4x=15+2x.** • Solve $8.5 < 3x + 2 \leq 9.7$ and graph the solution on the number line.
A1.4.B Write and graph an equation for a line given any of the following: the slope and the y-intercept, the slope and a point on the line, or two points on the line, and translate between these forms of linear equations. **(A1.2.C and A1.2.D)**	A related concept is that two parameters are sufficient to determine a line. Examples: • Find an equation for a line with y-intercept equal to 2 and slope equal to 3. • Find an equation for a line with slope 2 that goes through the point (1,1). • Find an equation for a line that goes through the points (-3,5) and (6,-2). • Describe the graph of y=2x+3 without plotting points or sketching the graph. • For each of the following, use only the equation (without sketching the graph) to describe the graph. • $y = 2x + 3$ • $y - 7 = 2(x - 2)$ • A linear function can be expressed in the following two ways: $y = 2x + 3$ $-6 = 4x - 2y$ • **Write the equation 3x+2y=5 in slope intercept form.**

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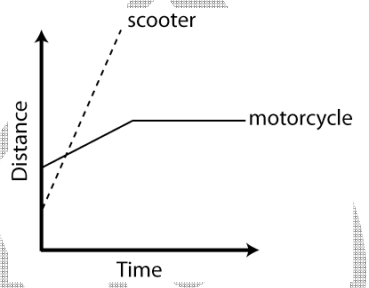
	• Write the equation y-1=2(x-2) in slope intercept form.
A1.4.C Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines. **(A1.2.E)**	**Students learn that the x-intercept for the function y=mx+b is the solution to the equation 0=mx+b. They understand that tables and graphs do not provide exact values in all situations.** Examples: • The graph shows the relationship between time and distance from a gas station for a motorcycle and a scooter. Make up a story about the motorcycle and scooter that matches the information in the graph. How are the slope and *y*-intercept of the line for the scooter represented in the situation?  • A 1,500-gallon tank contains 200 gallons of water. Water begins to run into the tank at the rate of 75 gallons per hour. When will the tank be full? Write a linear function that models this situation, draw a graph, and create a table of data points. Once you have answered the question and completed the tasks, explain your reasoning. Interpret the slope and *y*-intercept of the function in the context of the situation. • Need example about parallel / perpendicular- Build a square, calculate the slope
A1.4.D Write and solve systems of two linear equations and inequalities in two variables. **(A1.3.G)**	Example: • Two plumbing companies charge different rates for their service. Clyde's Plumbing Company charges a $75-per-visit fee that includes one hour of work plus $45 dollars per hour after the first hour. We-Unclog-It Plumbers charge a $100-per-visit fee that includes one hour of work plus $40 per hour after the first hour. For how many hours of plumbing work would Clyde's be less expensive than We-Unclog-It? • **Need example of word problem.** • Solve the following simultaneous linear equations algebraically:

	• -2x+y=2 • x+y= -1 Graph the above two linear equations. **Does this need a note that it is not continuous or does the language need to change to make it clear that it is?**
A1.4.E Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent. **(A1.3.B)**	In the case of a linear function $y = f(x)$, expressed in slope-intercept form ($y = mx + b$), m and b are parameters. Students should know that $f(x) = kx$ represents a direct variation (proportional relationship). Examples: • Graph a function of the form f(x) = kx, describe the effects that changes on k and x have on the graph and on f(x), and answer questions that arise in situations modeled proportional relationships. • **This problem needs rewording so it is clear it is a continuous linear function.** Laurie invests in a no-interest checking account with an initial deposit of $100, adding $30 per month and making no withdrawals. Write a function that represents this situation and then graph the function. If the rate of monthly deposits changes from $30 to $50, how will the graph of the function change? If the initial deposit changes from $100 to $200, how will the graph of the function change? • Compare and contrast the functions $y = 3\|x\|$ and $y = \frac{1}{3}\|x\|$.

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A1.5. Core Content: Quadratic functions and equations
(Algebra)

The introductory language should be rewritten similar to the following to reflect the change in content in this section.

Students study quadratic functions, their graphs, and quadratic equations in Algebra I. They use quadratic functions in the real number system to represent and model problems and answer questions in situations that are modeled by these non-linear functions. Algebra I focuses on real roots of quadratic equations, while in Algebra II studied more thoroughly using complex numbers. The important mathematical technique of completing the square is developed enough so that the quadratic formula can be derived. This is not the only type of non-linear function included in the Algebra I standards, but it is an area of focus.

Revised Expectation	Comments/Examples
A1.5.A Write and represent a quadratic function with a symbolic expression, graph it, make a table using it, give a verbal description of it, and make connections among these representations. **(A1.4.E)**	**The language from the *Comments* is now included in the *Expectation*.** **An Example is needed.**

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A1.5.B Sketch the graph of a quadratic function of the form y = ax^2 + b, describe the effects that changes in the parameters a and b have on the graph, and interpret the x-intercepts as solutions to a quadratic equation. **(A1.4.F)**	Example: • A quadratic function can be expressed in the following two ways: $f(x) = -(x-3)^2 + 1$ $f(x) = -(x-2)(x-4)$ What information about the graph can be directly inferred from each of these forms? Explain your reasoning. • Sketch the graph of y= $x^2 - 1$, showing the roots. Parameters include: • *a* nd *c* in $f(x) = ax^2 + c$ • *a*, *h*, and *k* in $f(x) = a(x-h)^2 + k$ and • *r*, *s*, and *a* in $f(x) = a(x-r)(x-s)$ Note that in Alg I, the parameter b in the term bx in the quadratic form $ax^2 + bx + c$ does not provide useful information about the characteristics of the graph.
A1.5.C Solve quadratic equations that can be factored as (ax+b)(cx+d) where a,b,c, and d are integers, recognizing special quadratics as squares and differences of squares. **(A1.1.H)**	**Students learn to efficiently solve quadratic equations by recognizing and using the simplest factoring methods.** **WE NEED EXAMPLES THAT SOLVE QUADRATIC EQUATIONS, NOT JUST FACTOR** **Examples:** • $2x^2 + x - 3 = (x-1)(2x+3)$ • $4x^2 + 6x = 2x(2x+3)$ • **(ax +b)^2 = a^{2}x^2 + 2abx + b^2 and (ax + b)(ax – b) = a^{2}b^2 – b^2** • $36x^2 - 25y^2 = (6x+5y)(6x-5y)$ • $x^2 + 6xy + 9y^2 = (x+3y)^2$

Draft

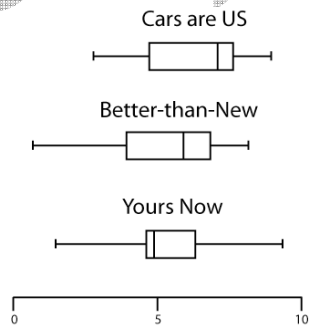
A1.5.D Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.
 	
(New)
	For those equations that are not easily factored, students solve them by completing the square and by using the quadratic formula. Completing the square should also be used to derive the quadratic formula.
	
	They learn how to determine if there are 2 real solutions, one real solution, or none.
	
	Students can **also** approximate solutions using graphs and tables.
	
	Example:
	• **Find x when 0=** $x^2 - 8$**. Answer: x=**$\pm 2\sqrt{2}$**.**
	• **Find x when 0=** $2x^2 + x - 3 = (x-1)(2x+3)$**.**
	  • Answer: x=1 and -3/2.
	
	Examples:
	• $4x^2 - 2x - 5 = 0$; $4x^2 - 2x = 5$;
	$x^2 - \frac{1}{2}x = \frac{5}{4}$;
	$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{5}{4} + \frac{1}{16}$
	$\left(x - \frac{1}{4}\right)^2 = \frac{21}{16}$; $x - \frac{1}{4} = \pm\sqrt{\frac{21}{16}} = \pm\frac{\sqrt{21}}{4}$
	$x = \frac{1}{4} \pm \frac{\sqrt{21}}{4} = \frac{1 \pm \sqrt{21}}{4}$
	x ≈ -0.9 and x ≈ 1.4
	
	• **An example is needed for quadratic formula:**
	  • Problem about intercepts

A1.6. Core Content: **Data and distributions**

(Data/Statistics/Probability)

Students select mathematical models for data sets and use those models to represent, describe, and compare data sets. They analyze the relationship between two variables and make and defend appropriate predictions, conjectures, and generalizations based on data. Students understand limitations of conclusions based on results of a study or experiment and recognize common misconceptions and misrepresentations.

Revised Expectation	Comments/Examples
A1.6.A Select, evaluate and calculate summary statistics for univariate	This expectation extends what students learn in earlier grades to include standard deviation and the processes of evaluation and

data: measures of center, and measures of spread (interquartile range, range, variance and standard deviation.	justification. Students should evaluate the appropriateness of measure of center and spread and use them to accurately compare data sets. Given a data set, students can be asked which measures support a particular interpretation.
(A1.5.A)	**Data can be interpreted from different points of views.** Example: • The local minor league baseball team has a salary dispute. Players claim they are being underpaid, but managers disagree. Bearing in mind that a few top players earn salaries that are quite high, would it be in the managers' best interest to use the mean or median when quoting the "average" salary of the team? Why? What about the players? • Each box-and-whisker plot shows the prices of used cars (in thousands of dollars) advertised for sale at three different car dealers. If you want to go to the dealer whose prices seem least expensive, which dealer would you go to? Use statistics from the displays to justify your answer. • Which is more likely to be affected by an outlier in a set of data, the interquartile range or the standard deviation? 

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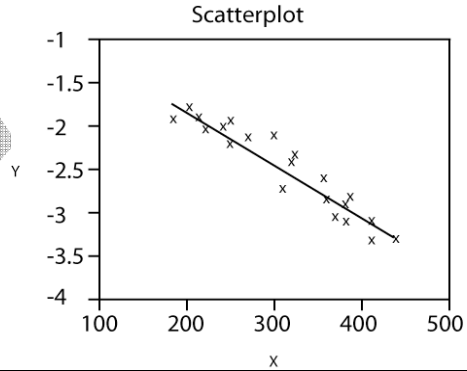
A1.6.B Describe how linear transformations affect the center and spread of univariate data. **(A1.5.B)**	A univariate set of data identifies data on a single variable, such as shoe size. Examples: • A company decides to give every one of its employees a $5,000 raise. What happens to the mean and standard deviation of the salaries as a result? • A company decides to double each of its employee's salaries. What happens to the mean and standard deviation of the salaries as a result?
A1.6.C Find the equation of the linear function that best fits data that are linearly related, interpret the slope and y-intercept of the line and use the equation to make predictions. **(A1.5.C)**	A bivariate set of data presents data on two variables, such as shoe size and height. In high school, the emphasis is on using a line of best fit to interpret data. Students can use technology to obtain a line of best fit.
A1.6.D Describe the correlation of data in scatter plots in to the linear function terms of strong or weak and positive or negative. **(A1.4.E)**	Examples: • Which words—*strong*, *weak*, *positive*, or *negative*—could be used to describe the relationship shown in the sample scatterplot below? 

A1.7. Additional Key Content: ***(Algebra)***

Language about other exponential functions and arithmetic sequences needs to be added here.

Revised Expectation	Comments/Examples
A1.7.A Plot the graph of an exponential function of the form $y = ab^n$ where n is an integer; describe the effects that changes in the parameters *a* and *b* have on the graph, and answer questions that arise in situations modeled by exponential functions. **(A1.4.A)**	Examples: • Plot the graph of $y = 2^n$ by hand. • You have won a door prize and are given a choice between two options: • a. $150 invested for 10 years at 4% compounded annually. • b. $200 invested for 10 years at 3% compounded annually. How much is each worth at the end of each year of the investment? Are the two investments ever equal in value? Which will you choose?
A1.7.B Approximate the solution to an exponential equation. **(A1.4.B)**	Students can approximate solutions using graphs or tables with or without technology.
A1.7.C Express arithmetic sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence. **(A1.3.F)**	Examples: • Write a recursive formula for the arithmetic sequence 5, 9, 13, 17, ... What is the slope of the line that contains the points associated with these values and their position in the sequence? How is the slope of the line related to the sequence? • Given that $u(0) = 3$ and $u(n + 1) = u(n) + 7$ when n is a positive integer, a. find $u(5)$; b. find n so that $u(n) = 361$; and c. find a formula for $u(n)$.
A1.7.D Solve an equation involving several variables by expressing one variable in terms of the others. **(A1.1.I)**	Examples: • Solve $A = p + prt$ for p. • Solve $V = \pi r^2 h$ for h or for r.

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A1.8. Core Processes: ***Reasoning, problem solving, and communication***

Students formalize the development of reasoning in high school as they make and defend generalizations and justify their reasoning using accepted standards of mathematical evidence and proof. Students extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to practical and applied situations. Students use correct mathematical language, terms, and symbols as they address problems in Algebra 1. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an accelerating amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Revised Expectation	Comments/Examples
A1.8.A Use algebra and the properties of number systems to develop valid mathematical arguments, make and prove conjectures, and find counterexamples to refute false statements. We agreed to move this to process. (A1.1.E) **A1.8.B Analyze a problem situation, determine the question(s) to be answered, synthesize given information, determine how to represent the problem mathematically, and identify implicit and explicit assumptions that have been made.** **A1.8.C Solve problems by formulating one or more strategies, applying the strategies, and verifying the solutions.**	This builds on previous work students have done to make and build conjectures. Students will build on their understanding as they formalize proof in later courses. Examples: • Prove $(a + b)^2 = a^2 + 2ab + b^2$. • A student writes $(x + 3)^2 = x^2 + 9$. Explain why this is incorrect. **Also need examples that are more challenging—insert number theory examples** Descriptions of solution processes, explanations, and justifications can include numbers, words (including mathematical language), pictures, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

A1.8.D Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.

A1.8.E Generalize a solution strategy for a single problem to a class of related problems.

A1.8.F Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.

A1.8.G Summarize and communicate mathematical ideas and reasoning with precision and efficiency in ways appropriate for the intended audience and purpose.

A1.8.H Synthesize mathematical information from multiple sources to draw a conclusion, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning.

A1.8.I Use inductive reasoning to make conjectures, use deductive reasoning to prove a valid conjecture, and develop a counterexample to refute an invalid conjecture.

A1.8.J Explain and demonstrate the importance of generalizations in mathematics and the role of generalizations in inductive and deductive reasoning.

Examples:

- Three teams of students independently conducted experiments to relate the rebound height of a ball to the rebound number. The table gives the average of the teams' results.

Rebound Number	Rebound Height (cm)
0	200
1	155
2	116
3	88
4	66
5	50
6	44

  Construct a scatterplot of the data, and describe the function that relates the height of the ball to the rebound number. Predict the rebound height of the ball on the tenth rebound. Justify your answer.

- **A scale model of the Statue of Liberty is 1.5 inches tall. The scale is 1 inch to 200 feet. How tall is the Statue of Liberty?**

Hamburger:
8 pounds for $10

- **How much is the hamburger per pound?**
- **How much would 6 pounds of hamburger cost?**
- **How much hamburger can you buy for $2?**

Algebra I – Version 2

Students learn to solve many new types of problems in Algebra I. They are introduced to several types of functions, spending considerable time with linear and quadratic functions. In addition to exploring the idea of functions, they learn that specific functions model contextual situations captured in word problems and can be used to solve those word problems. Each new function allows students to solve another class of problems. Many problems that appear, on the surface, to be very different from each other can be represented by identical functions? Equations? . This is the beauty and unifying principle of algebra, i.e. that the same algebraic techniques can be applied to a wide variety of different situations.

Deciding what kind of approach best models a particular situation and representing word problems with the corresponding algebraic information is nontrivial mathematical content in itself.

A1.1. Core Content: ***Numbers, expressions, and operations***
(Numbers, Operations, Algebra)

Students see the number system extended to the real numbers represented by the number line. They work with integer exponents, scientific notation and radicals. They use variables and expressions to solve problems from both purely mathematical and applied contexts. They extend their understanding of arithmetic operations and properties, enabling them to write and manipulate a wide variety of algebraic expressions. They will demonstrate this understanding throughout high school mathematics as they apply algebraic procedures to solve problems modeled by functions.

Expectation	**Comments/Examples**
A1.1.A Explain the relationship between real numbers and the number line, compare and order real numbers with and without the number line. **(A1.1.A)**	**Students learn that every point on the number line represents a real number, either rational or irrational, and that every real number has its unique point on the number line. They locate, compare, and order the set of real numbers on the number line.** Real numbers include those written in scientific notation or expressed as fractions, decimals, or roots. Examples: • Without using a calculator, order the following on the number line: $\sqrt{82}$, 3π, 8.9, 9, $\frac{37}{4}$, 9.3×10^0

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- A star's color gives an indication of its temperature and age. The chart shows four types of stars and the lowest temperature of each type.

Type	Lowest Temperature (in °F)	Color
A	1.35×10^4	Blue-White
B	2.08×10^4	Blue
G	9.0×10^3	Yellow
P	4.5×10^4	Blue

List the temperatures in order from lowest to highest.

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A1.1.B Use and interpret letters for different purposes in algebraic expressions and equations.

(A1.1.C)

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Letters can be used:

- to represent fixed and temporarily unknown values in equations (such as $3x + 2 = 5$);
- to express identities (such as $x + x = 2x$ for all x);
- as attributes in formulas (such as $A = lw$);
- as parameters in equations (such as the m and b for the family of functions defined by $y = mx + b$); and
- as names of universal constants, such as π;
- to represent varying quantities (such as x in $f(x) = 5x$)
- **to represent functions (such as f in $f(x) = 5x$).**

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A1.1.C Evaluate algebraic expressions when variables are assigned specific values and determine all possible values of a variable in a given algebraic expression that assure the expression satisfies a prescribed condition.

(A1.1D and A1.1.F)

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Expressions include those involving polynomials, radicals, absolute values, and integer exponents.

Examples:

- For what values of a is $\frac{1}{a}$ an integer?
- For what values of a is $\sqrt{5 - a}$ **defined?**
- For what values of a is $-a$ always positive?

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A1.1.D Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to translate among equivalent forms of numeric and algebraic expressions. **(A1.1.G)**	Examples: • $\frac{2^{-2}3^2 5}{2^2 3^{-3} 5^2} = \frac{3^5}{2^4 5}$ • $\frac{a^{-2}b^2 c}{a^2 b^{-3} c^2} = \frac{b^5}{a^4 c}$ • $\sqrt{8} = \sqrt{2*2*2} = 2\sqrt{2}$ • $\sqrt[3]{a \bullet b} = \sqrt[3]{a} \bullet \sqrt[3]{b}$ • **Need an interpretation example**
A1.1.E Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection. **(A1.1.B)**	**Students need to determine whether approximate or exact values of real numbers are appropriate depending on the necessary degree of precision needed in given situations. They also need to decide, after calculating or estimating, if the amount of error is within tolerable limits.** Decimal approximations of numbers are sometimes used in applications such as carpentry or engineering, while in mathematics, exact values may be required. Students should understand the difference. For example, 1.414 is an approximation and not an exact solution to the equation $x^2 - 2 = 0$, but $\sqrt{2}$ is an exact solution to this equation. Example: • **Find the length of the diagonal of a rectangular room that is 12 feet by 14 feet. Give your answer to the nearest inch.** • Using a common engineering formula, an engineering student represented the maximum safe load of a bridge to be $1000(99 - 70\sqrt{2})$ tons. He used 1.41 as the approximation for $\sqrt{2}$ in his calculation. When the bridge was built and tested in a computer simulation to verify its maximum weight-bearing load, it collapsed! The student was shocked, as he had estimated that the bridge would hold ten times the weight that was applied to it when it collapsed. Calculate the weight that the student thought the bridge could bear using 1.41 as the estimate for $\sqrt{2}$.

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	Calculate other weight values using estimates of $\sqrt{2}$ that have more decimal places. What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.
A1.1.F Write algebraic expressions in equivalent forms by using algebraic properties to factor and combine like terms. **(A1.1.H)**	Algebraic properties include the commutative, associative, and distributive properties. Factoring includes: • factoring a monomial from a polynomial: $4x^2 + 6x = 2x(2x + 3)$; • factoring the difference of two squares, such as $36x^2 - 25y^2 = (6x + 5y)(6x - 5y)$ and $x^4 - y^4 = (x + y)(x - y)(x^2 + y^2)$; • factoring perfect square trinomials, such as $x^2 + 6xy + 9y^2 = (x + 3y)^2$; • factoring quadratic trinomials that have a leading coefficient of 1 and factors with integer coefficients, such as $x^2 + 5x + 4 = (x + 4)(x + 1)$; and • factoring trinomials that can be expressed as the product of a constant and a trinomial, such as $2x^2 - 10x - 28 = 2(x^2 - 5x - 14)$ and $0.5x^2 - 2.5x - 7 = 0.5(x^2 - 5x - 14)$.
A1.1.G Write algebraic expressions in equivalent forms by using algebraic properties to perform the four arithmetic operations with polynomials **(A1.1.H)**	Algebraic properties include the commutative, associative, and distributive properties. Students should recognize that expressions are essentially sums, products, differences, or quotients. For example, **the sum $2x^2 + 4x$ can be written as a product, $2x(x + 2)$.** Examples: • $\frac{2x^2}{9} \bullet \frac{6}{2x^4}$ • **More examples are needed that are intermediate in level of difficulty**

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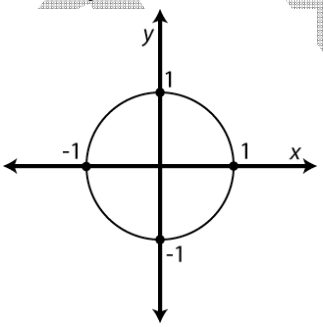
A1.2. Core Content: ***Types and behaviors of functions***
(Algebra)

Students formalize and deepen their understanding of functions, their defining characteristics and uses, and the mathematical language used to describe functions. They learn that functions are often specified by an equation of the form $y = f(x)$, where any allowable x-value yields a unique y-value. Students study linear and quadratic functions. They also learn about absolute value functions, exponential functions, and those that can be defined piecewise. In every case students learn about the representations and basic transformations of these functions and the practical and mathematical limitations that must be considered when working with functions or when using functions to model situations.

Expectation	Comments/Examples
A1.2.A Select and justify a function to model a situation, describe appropriate domain restrictions; use the function to solve the problem and interpret the solution in context of the original situation. **(A1.2.F)**	Functions include linear, quadratic, exponential, and those defined piecewise (including functions that contain the absolute value of an expression and step functions). Analyze the rate of change of a function represented by a table or graph to determine whether the function might be linear, and analyze common ratios to determine whether the function might be exponential. Common ratios in an exponential function are determined by calculating the ratio of consecutive entries of the dependent variable in a table corresponding to equal-sized increments of the independent variable. Example: Anderson Lake is a popular lake to catch trout. In 1998, the lake was stocked with 10,000 fish. In the next five years, 15% of the fish were caught by anglers and removed. c. Write a function that models the fish population size at the end of the fishing season each year. Justify the function you wrote. d. State and appropriate domain for the model based on the context.
A1.2.B Identify functions, their domains, ranges, roots and independent and dependent variables. **(A1.2.D)**	A function expresses dependence between two quantities, one of which is independent and the other of which is dependent. Functions include linear, quadratic, exponential, and those defined piecewise (including functions that contain the absolute value of an expression and step functions). Given a problem situation, students should describe further restrictions on the domain of a function that are appropriate for the problem context.

	Example:
	• A function $f(n) = 60n$ is used to model the distance in miles traveled by a car traveling 60 miles per hour in n hours. Identify the domain and range of this function. What restrictions on the domain of this function should be considered for the model to correctly reflect the situation?
	• What is the domain of $f(x) = \sqrt{5 - x}$?
	• Which of the following equations, inequalities, or graphs determine y as a function of x?
	  • $y = 2$
	  • $x = 3$
	  • $y = \|x\|$
	  • $x = \|y\|$, where $y > 0$
	  • $x = \|y\|$, where x is a real number
	  • $x^2 + y^2 = 1$
	 
	
	• **Which of the following are functions? Explain why or why not.**
	  • **The age in years of each student in your math class and each student's shoe size**
	  • **The letters associated with the numerals on a telephone**

A1.2.C Represent a function, f(x), as a graph, determine if a graph represents a function. **(A1.2.E)**	Examples: • Describe key characteristics of the graph of $f(x) = \|x - 3\| + 5$.

A1.2.D Represent a function with a symbolic expression, as a graph, in a table, give a verbal description of it, and make connections among these representations. (A1.2.B)	This expectation applies each time a new class (family) of functions is encountered. **Students should be introduced to a variety of functions, including x^3, $\sqrt{x}$, $\frac{1}{x}$ and absolute value functions such as f(x)=|x-3|+5. They should know that f(x) = $\frac{a}{x}$ represents an inverse proportion.** **Translating among these various representations of functions is an important way to demonstrate an conceptual understanding of functions.** Representations include • a symbolic expression that assigns a unique value to each element of a domain, • a table that exhibits a correspondence explicitly for some values of the domain, • a graph in the coordinate plane, and • a verbal description. **Students should know** that a table of values may not uniquely determine a single function without some specification of the nature of that function (e.g., it is quadratic).

A1.2.E Evaluate *f*(*x*) at *a* (i.e., *f*(*a*)), solve for *x* in the equation *f*(*x*) = *b*, and distinguish between the two.
 	
(A1.2.C)
	Functions may be described and evaluated with symbolic expressions, tables, graphs, or verbal descriptions.
	
	Example:
	• Roses-R-Red sells its roses for $0.75 per stem and charges a $20 delivery fee per order.
	  • What is the cost of having 10 roses delivered?
	  • How many roses can you have delivered for $65?

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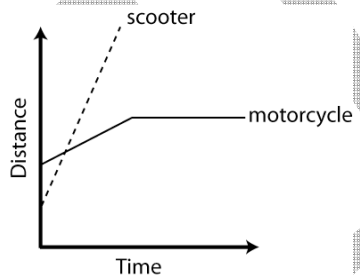
A1.3. Core Content: ***Linear functions, equations, and inequalities***
(Algebra)

Students understand that linear functions can be used to model situations involving a constant rate of change. They use linear functions to analyze relationships, represent and model problems, and answer questions. They answer these questions by writing and solving equations using algebraic techniques, and extend these techniques to other types of equations. Students also learn graphical and numerical methods for approximating solutions to equations. Students interpret the meaning of problem solutions and explain their limitations. These algebraic skills are applied in other Core Content areas across high school courses.

Revised Expectation	Comments/Examples
A1.3.A Solve problems that can be modeled by one or two linear equations or inequalities and interpret the solutions in the context of the problems. **A1.3.G)**	Examples: • The assistant pizza maker makes 6 pizzas an hour. The master pizza maker makes 10 pizzas an hour but starts two hours later than his assistant. They must make 92 pizzas. How many hours from when the assistant starts will it take? **What is a general form of the equation that could be used to determine the number of pizzas for any number of hours (h)?** • A bird flies 5 miles in 20 minutes into the wind but takes only 10 minutes to fly back with the wind. The bird always flies at a constant rate with respect to the air. How many miles per hour does the bird fly and what is the wind speed in miles per hour? **Need two or three additional problems to illustrate the type of situations linear functions model.**
A1.3.B Write and solve linear equations and inequalities in one variable. **(A1.6.A)**	This expectation includes the use of absolute values in the equations and inequalities. Examples: • A cup is 6 cm tall, including a 1.1 cm lip. Find a formula that gives the height of a stack of cups in terms of the number of cups in the stack. Find a formula for the number of cups for a given height. Create general forms of your formulas that would work for any cup with overall height *h* and lip height *c*. • Joe and Sue sell grape, cherry and lime snow cones in proportions 2:3:5. They start the day with 80 fluid ounces of syrup. How much of each kind should they take?

	• Maddie is watching cars go by on the road in front of her house. She notices that one car goes by every second. She knows that the cars are going 30 miles per hour.
	  • How many feet apart are the cars?
	  • How many cars are on one mile of roadway?
	• Write an absolute value equation or inequality for
	  • All the numbers 2 units from 7, and
	  • All the numbers that are more than *b* units from *a*.
	• Solve $\|x - 6\| \leq 4$ and graph the solution on the number line.
	• Solve for x when 3x-5=4x+9.
	• Write an equation or inequality that has
	  • No real solutions
	  • Infinite numbers of real solutions
	  • Exactly one real solution
	• **Solve for x when 5 < |x+3| < 10.**
	  (Answer: The intervals 2 < x < 7 and -13 < x < -8)
	• **Solve for x in 2(x-3)+4x=15+2x.**
	• Solve $8.5 < 3x + 2 \leq 9.7$ and graph the solution on the number line.

A1.3.C Write and graph an equation for a line given any of the following: the slope and the y-intercept, the slope and a point on the line, or two points on the line, and translate between these forms of linear equations.
 	
(A1.2.C and A1.2.D)
	A related concept is that two parameters are sufficient to determine a line.
	
	Examples:
	• Find an equation for a line with y-intercept equal to 2 and slope equal to 3.
	• Find an equation for a line with slope 2 that goes through the point (1,1).
	• Find an equation for a line that goes through the points (-3,5) and (6,-2).
	• Describe the graph of y=2x+3 without plotting points or sketching the graph.
	• For each of the following, use only the equation (without sketching the graph) to describe the graph.
	  • $y = 2x + 3$
	  • $y - 7 = 2(x - 2)$
	• A linear function can be expressed in the following two ways:

	$y = 2x + 3$ $-6 = 4x - 2y$ • **Write the equation 3x+2y=5 in slope intercept form.** • Write the equation y-1=2(x-2) in slope intercept form.
A1.3.D Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines. **(A1.2.E)**	**Students learn that the x-intercept for the function y=mx+b is the solution to the equation 0=mx+b. They understand that tables and graphs do not provide exact values in all situations.** Examples: • The graph shows the relationship between time and distance from a gas station for a motorcycle and a scooter. Make up a story about the motorcycle and scooter that matches the information in the graph. How are the slope and *y*-intercept of the line for the scooter represented in the situation?  • A 1,500-gallon tank contains 200 gallons of water. Water begins to run into the tank at the rate of 75 gallons per hour. When will the tank be full? Write a linear function that models this situation, draw a graph, and create a table of data points. Once you have answered the question and completed the tasks, explain your reasoning. Interpret the slope and *y*-intercept of the function in the context of the situation. • Need example about parallel / perpendicular- Build a square, calculate the slope
A1.3.E Write and solve systems of two linear equations and inequalities in two variables. **(A1.3.G)**	Example: • Two plumbing companies charge different rates for their service. Clyde's Plumbing Company charges a $75-per-visit fee that includes one hour of work plus $45 dollars per hour after the first hour. We-Unclog-It Plumbers charge a $100-per-visit fee that includes one hour of work plus $40 per hour after the first hour. For how many hours of plumbing work would Clyde's be less expensive than We-Unclog-It? • **Need example of word problem.**

	• Solve the following simultaneous linear equations algebraically:
	  • -2x+y=2
	  • x+y= -1
	
	Graph the above two linear equations. **Does this need a note that it is not continuous or does the language need to change to make it clear that it is?**

A1.3.F Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent. **(A1.3.B)**	In the case of a linear function $y = f(x)$, expressed in slope-intercept form ($y = mx + b$), m and b are parameters. Students should know that $f(x) = kx$ represents a direct variation (proportional relationship). Examples: • Graph a function of the form f(x) = kx, describe the effects that changes on k and x have on the graph and on f(x), and answer questions that arise in situations modeled proportional relationships. • **This problem needs rewording so it is clear it is a continuous linear function.** Laurie invests in a no-interest checking account with an initial deposit of $100, adding $30 per month and making no withdrawals. Write a function that represents this situation and then graph the function. If the rate of monthly deposits changes from $30 to $50, how will the graph of the function change? If the initial deposit changes from $100 to $200, how will the graph of the function change? • Compare and contrast the functions $y = 3\|x\|$ and $y = \frac{1}{3}\|x\|$.

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A1.4. Core Content: ***Quadratic functions and equations***
(Algebra)

The introductory language should be rewritten similar to the following to reflect the change in content in this section.

Students study quadratic functions, their graphs, and quadratic equations in Algebra I. They use quadratic functions in the real number system to represent and model problems and answer questions in situations that are modeled by these non-linear functions. Algebra I focuses on real roots of quadratic equations, while in Algebra II studied more thoroughly using complex numbers. The important mathematical technique of completing the square is developed enough so that the quadratic formula can be derived. This is not the only type of non-linear function included in the Algebra I standards, but it is an area of focus.

Revised Expectation	Comments/Examples

A1.4.A Solve problems that can be modeled by quadratic equations and interpret the solutions in the context of the problems.
 	
(New)
	Examples:
	• Find the solution where the graphs of y=x+2 and y= x^2 intersect.
	  • Solution: (-1,1) and (2,4).
	• If you throw a ball straight up at 10 feet per second, how long before it comes back to you?
	• Solve for x in $(x-3)^2 = 16$.
	• How long does it take a ball to drop 100 feet?
	• Joe owns a small plot of land 20 feet by 30 feet. He wants to double the area by increasing both the length and the width, but keep them in the same proportion. What will be the new length and width?
	• What 2 consecutive numbers, when multiplied together give the first number plus 16?
	  • Solution: x(x+1)=x+16, so $0 = x^2 - 16 = (x-4)(x+4)$, so there are two solutions: {-4,-3} and {4,5}.

A1.4.B Write and represent a quadratic function with a symbolic expression, graph it, make a table using it, give a verbal description of it, and make connections among these representations. **(A1.4.E)**	**The language from the *Comments* is now included in the *Expectation.*** **An Example is needed.**

A1.4.C Sketch the graph of a quadratic function of the form y = ax^2 + b, describe the effects that changes in the parameters a and b have on the graph, and interpret the x-intercepts as solutions to a quadratic equation. **(A1.4.F)**	Example: • A quadratic function can be expressed in the following two ways: $f(x) = -(x-3)^2 + 1$ $f(x) = -(x-2)(x-4)$ What information about the graph can be directly inferred from each of these forms? Explain your reasoning. • Sketch the graph of y= $x^2 - 1$, showing the roots. Parameters include: • *a* nd *c* in $f(x) = ax^2 + c$ • *a*, *h*, and *k* in $f(x) = a(x-h)^2 + k$ and • *r*, *s*, and *a* in $f(x) = a(x-r)(x-s)$ Note that in Alg I, the parameter b in the term bx in the quadratic form $ax^2 + bx + c$ does not provide useful information about the characteristics of the graph.
A1.4.D Solve quadratic equations that can be factored as (ax+b)(cx+d) where a,b,c, and d are integers, recognizing special quadratics as squares and differences of squares. **(A1.1.H)**	**Students learn to efficiently solve quadratic equations by recognizing and using the simplest factoring methods.** **The examples should show solving quadratic equations, not just factoring.** **Examples:** • $2x^2 + x - 3 = (x-1)(2x+3)$ • $4x^2 + 6x = 2x(2x+3)$ • **(ax +b)^2 = a^{2}x^2 + 2abx + b^2 and (ax + b)(ax – b) = a^{2}b^2 – b^2** • $36x^2 - 25y^2 = (6x+5y)(6x-5y)$ • $x^2 + 6xy + 9y^2 = (x+3y)^2$

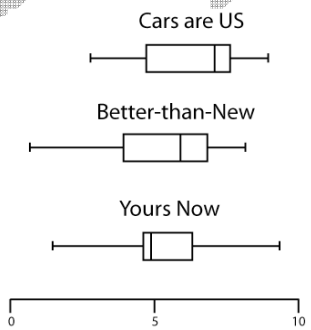
Draft

A1.4.E Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.
 	
(New)
	For those equations that are not easily factored, students solve them by completing the square and by using the quadratic formula. Completing the square should also be used to derive the quadratic formula.
	
	They learn how to determine if there are 2 real solutions, one real solution, or none.
	
	Students can **also** approximate solutions using graphs and tables.
	
	Example:
	• **Find x when 0=** $x^2 - 8$**. Answer: x=**$\pm 2\sqrt{2}$ **.**
	• **Find x when 0=** $2x^2 + x - 3 = (x-1)(2x+3)$**.**
	  • Answer: x=1 and -3/2.
	
	Examples:
	• $4x^2 - 2x - 5 = 0;\ 4x^2 - 2x = 5;$
	$x^2 - \frac{1}{2}x = \frac{5}{4};$
	$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{5}{4} + \frac{1}{16}$
	$\left(x - \frac{1}{4}\right)^2 = \frac{21}{16};\ x - \frac{1}{4} = \pm\sqrt{\frac{21}{16}} = \pm\frac{\sqrt{21}}{4}$
	$x = \frac{1}{4} \pm \frac{\sqrt{21}}{4} = \frac{1 \pm \sqrt{21}}{4}$
	x ≈ -0.9 and x ≈ 1.4
	
	• **An example is needed for quadratic formula:**
	  • Problem about intercepts

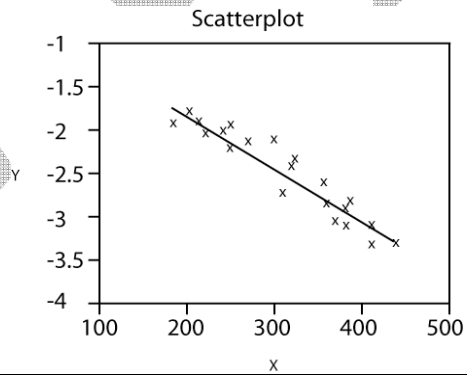
A1.5. Core Content: ***Data and distributions***
(Data/Statistics/Probability)

Students select mathematical models for data sets and use those models to represent, describe, and compare data sets. They analyze the relationship between two variables and make and defend appropriate predictions, conjectures, and generalizations based on data. Students understand limitations of conclusions based on results of a study or experiment and recognize common misconceptions and misrepresentations.

Revised Expectation	Comments/Examples
A1.5.A Select, evaluate and calculate summary statistics for univariate data: measures of center, and	This expectation extends what students learn in earlier grades to include standard deviation and the processes of evaluation and justification.

measures of spread (interquartile range, range, variance and standard deviation.	Students should evaluate the appropriateness of measure of center and spread and use them to accurately compare data sets. Given a data set, students can be asked which measures support a particular interpretation.
(A1.5.A)	**Data can be interpreted from different points of views.** Example: • The local minor league baseball team has a salary dispute. Players claim they are being underpaid, but managers disagree. Bearing in mind that a few top players earn salaries that are quite high, would it be in the managers' best interest to use the mean or median when quoting the "average" salary of the team? Why? What about the players? • Each box-and-whisker plot shows the prices of used cars (in thousands of dollars) advertised for sale at three different car dealers. If you want to go to the dealer whose prices seem least expensive, which dealer would you go to? Use statistics from the displays to justify your answer. • Which is more likely to be affected by an outlier in a set of data, the interquartile range or the standard deviation? 

Draft

A1.5.B Describe how linear transformations affect the center and spread of univariate data. **(A1.5.B)**	A univariate set of data identifies data on a single variable, such as shoe size. Examples: • A company decides to give every one of its employees a $5,000 raise. What happens to the mean and standard deviation of the salaries as a result? • A company decides to double each of its employee's salaries. What happens to the mean and standard deviation of the salaries as a result?
A1.5.C Find the equation of the linear function that best fits data that are linearly related, interpret the slope and y-intercept of the line and use the equation to make predictions. **(A1.5.C)**	A bivariate set of data presents data on two variables, such as shoe size and height. In high school, the emphasis is on using a line of best fit to interpret data. Students can use technology to obtain a line of best fit.
A1.5.D Describe the correlation of data in scatter plots in to the linear function terms of strong or weak and positive or negative. **(A1.4.E)**	Examples: • Which words—*strong*, *weak*, *positive*, or *negative*—could be used to describe the relationship shown in the sample scatterplot below? 

Draft

A1.6. Additional Key Content: ***Algebra***

Language about other exponential functions and arithmetic sequences needs to be added here.

Revised Expectation	Comments/Examples
A1.6.A Solve problems that that can be modeled by exponential functions and interpret solutions in the context of the problems. (new)	Two or Three example problems need to be added
A1.6.B Plot the graph of an exponential function of the form $y = ab^n$ where n is an integer; describe the effects that changes in the parameters *a* and *b* have on the graph, and answer questions that arise in situations modeled by exponential functions. **(A1.4.A)**	Examples: • Plot the graph of $y = 2^n$ by hand. • You have won a door prize and are given a choice between two options: • a. $150 invested for 10 years at 4% compounded annually. • b. $200 invested for 10 years at 3% compounded annually. How much is each worth at the end of each year of the investment? Are the two investments ever equal in value? Which will you choose?
A1.6.C Approximate the solution to an exponential equation. **(A1.4.B)**	Students can approximate solutions using graphs or tables with or without technology.

Draft

A1.6.D Express arithmetic sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence. **(A1.3.F)**	Examples: • Write a recursive formula for the arithmetic sequence 5, 9, 13, 17, ... What is the slope of the line that contains the points associated with these values and their position in the sequence? How is the slope of the line related to the sequence? • Given that $u(0) = 3$ and $u(n + 1) = u(n) + 7$ when n is a positive integer, a. find $u(5)$; b. find n so that $u(n) = 361$; and c. find a formula for $u(n)$.
A1.6.E Solve an equation involving several variables by expressing one variable in terms of the others. **(A1.1.I)**	Examples: • Solve $A = p + prt$ for p. • Solve $V = \pi r^2 h$ for h or for r.

Draft

A1.7. Core Processes: ***Reasoning, problem solving, and communication***

Students formalize the development of reasoning in high school as they make and defend generalizations and justify their reasoning using accepted standards of mathematical evidence and proof. Students extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to practical and applied situations. Students use correct mathematical language, terms, and symbols as they address problems in Algebra 1. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an accelerating amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Revised Expectation	Comments/Examples
A1.7.A Use algebra and the properties of number systems to develop valid mathematical arguments, make and prove conjectures, and find counterexamples to refute false statements. We agreed to move this to process. (A1.1.E) **A1.7.B Analyze a problem situation, determine the question(s) to be answered, synthesize given information, determine how to represent the problem mathematically, and identify implicit and explicit assumptions that have been made.** **A1.7.C Solve problems by formulating one or more strategies, applying the strategies, and verifying the solutions.**	This builds on previous work students have done to make and build conjectures. Students will build on their understanding as they formalize proof in later courses. Examples: • Prove $(a + b)^2 = a^2 + 2ab + b^2$. • A student writes $(x + 3)^2 = x^2 + 9$. Explain why this is incorrect. **Also need examples that are more challenging—insert number theory examples** Descriptions of solution processes, explanations, and justifications can include numbers, words (including mathematical language), pictures, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

A1.7.D Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem.

A1.7.E Generalize a solution strategy for a single problem to a class of related problems.

A1.7.F Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.

A1.7.G Summarize and communicate mathematical ideas and reasoning with precision and efficiency in ways appropriate for the intended audience and purpose.

A1.7.H Synthesize mathematical information from multiple sources to draw a conclusion, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning.

A1.7.I Use inductive reasoning to make conjectures, use deductive reasoning to prove a valid conjecture, and develop a counterexample to refute an invalid conjecture.

A1.7.J Explain and demonstrate the importance of generalizations in mathematics and the role of generalizations in inductive and deductive reasoning.

Examples:

- Three teams of students independently conducted experiments to relate the rebound height of a ball to the rebound number. The table gives the average of the teams' results.

Rebound Number	Rebound Height (cm)
0	200
1	155
2	116
3	88
4	66
5	50
6	44

  Construct a scatterplot of the data, and describe the function that relates the height of the ball to the rebound number. Predict the rebound height of the ball on the tenth rebound. Justify your answer.

- **A scale model of the Statue of Liberty is 1.5 inches tall. The scale is 1 inch to 200 feet. How tall is the Statue of Liberty?**

Hamburger:
8 pounds for $10

- **How much is the hamburger per pound?**
- **How much would 6 pounds of hamburger cost?**
- **How much hamburger can you buy for $2?**

Geometry

G.1. Core Content: Logical arguments and proofs (Logic)

Consider language such as the following

Students formalize the reasoning skills they have developed in previous grades' work on mathematical processes. Geometry is the study of shape and space, but it introduces ideas of axioms and theorems and the concept of deductive proof based on them. Students solidify their understanding of what it means to prove a geometric statement mathematically. They use inductive reasoning to formulate conjectures about geometric relationships and use deductive reasoning to establish the truth of conjectures or reject them on the basis of counterexamples. Students explain their reasoning as they formulate, test, and prove or refute conjectures using precise mathematical language and symbols.

Expectations	Comments/Examples
G.1.A Distinguish between inductive and deductive reasoning and identify situations where each might be used. (new)	**Needs example such as the following:**
G.1.B Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample. (G.1.A)	Examples: • Investigate the relationship among the medians of a triangle using paper folding or dynamic geometry software. Make a conjecture about this relationship. • **Computer software could be used to see the strong likelihood that the theorems in G.3.A are true. This inductive reasoning is not a formal proof.**
G.1.C Use deductive reasoning to prove that a valid geometric statement is true. (G.1.B)	Valid proofs may be presented in paragraph, two-column, or flow-chart formats. **Proof by contradiction is a form of deductive reasoning.** Example: • Prove that the diagonals of a rhombus are perpendicular bisectors of each other.

G.1.D Write the converse, inverse, and contrapositive of a valid proposition and determine their validity. (G.1.B)	**Students judge arguments for their validity and give counterexamples for false statements.** Examples: • If *m* and *n* are odd integers, then the sum of *m* and *n* is an even integer. State the converse and determine whether it is valid. • If a quadrilateral is a rectangle, the diagonals have the same length. State the contrapositive and determine whether it is valid.
G.1.E Identify errors or gaps in a mathematical argument, and develop counterexamples to refute invalid statements about geometric relationships. (G.1.B)	Example: • Identify errors in reasoning in the following proof: • Given $\angle ABC \cong \angle PRQ$, $AB \cong PQ$, and $BC \cong QR$, then $\Delta ABC \cong \Delta PQR$ by SAS.

G.1. Core Content: ***Lines and Angles*** (Geometry/Measurement)

<u>This section needs development to reflect current content. Language similar to the following should be included.</u>

Students study basic properties of parallel and perpendicular lines, their respective slopes, and the properties of the angles formed when the parallel lines are cut by a transversal.

Expectation	Comments/Examples
G.2.A State, prove, and apply theorems about parallel and perpendicular lines. (G.2.A)	**Students should be able to summarize and explain the basic theorems.** Examples: • If a point is on the perpendicular bisector of a line segment, then the point is equidistant from the ends of the line segment. • If each of two lines is perpendicular to a given line, then the two lines are parallel

G.2.B State, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal. (G.2.B)	Example: If two parallel lines are cut by a transversal, then alternate-interior angles are equal. • Prove the angle sum theorem for triangles. • **New example that needs a diagram: Take two parallel lines with point A on one and C and D on the other. Draw a line segment between C and A and another line segment with the same length to locate B on the line with A. Prove that BC bisects the angle ACD**.
G.2.C Perform basic constructions with compass and straightedge related to parallel and perpendicular lines and explain why they work. (new)	**Using a virtual circle and line is equivalent to paper and pencil constructions.** Examples: • Bisect a line segment • Drop a perpendicular from a point to a line • Construct a line through a point that is parallel to another line.
G.2.D Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space. (G.2.K)	Example: • Describe all the ways that three planes can intersect in space.

Draft

G.3. Core Content: *Two- and three-dimensional figures*

(Geometry/Measurement)

Students know and can prove theorems about two- and three-dimensional geometric figures, both formally and informally. They identify necessary and sufficient conditions for proving congruence, similarity, and properties of figures. This leads to congruence as a special form of congruence, working with right triangles, and the Pythagorean theorem. Students also extend their learning to polygons, including classification, theorems regarding angle and side measures, diagonals, and other problems in both purely mathematical and applied contexts.

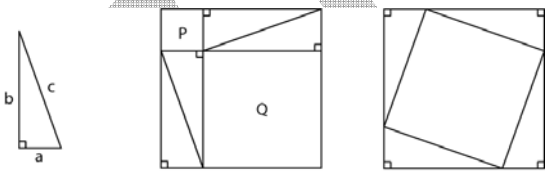
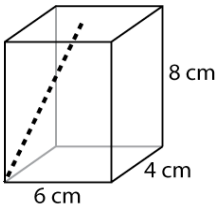
Expectation	Comments/Examples

G.3.A State, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.
 	
(new)
	Examples:
	• The sum of the angles of a triangle is 180°.
	• The bisectors of the angles of a triangle intersect in a point (the incenter) that is equidistant from the three sides of the triangle.
	• The perpendicular bisectors of the sides of a triangle intersect in a point (the circumcenter) that is equidistant from the three vertices of the triangle.
	• The lines that contain the altitudes of a triangle intersect in a point (the orthocenter).
	• The medians of a triangle intersect in a point (the centroid) that is two thirds of the distance from each vertex to the midpoint of the opposite side.
	• **Show two SSA triangles can be similar but not congruent.**
	• **Two triangles are congruent if:**
	  • **Three sides of the triangles are congruent**
	  • **Two angles and the included side are congruent**
	  • **Two sides and the included angle are congruent**
	  • **The hypotenuse and one leg of the right angle are congruent to the corresponding parts of the second right triangle**

Draft

G.3.B Determine and prove triangle congruence, triangle similarity, and other properties of triangles. (G.2.C)	**Students should identify necessary and sufficient conditions for congruence and similarity in triangles, and use these conditions in proofs.** Examples: • Prove that congruent triangles are similar. • For a given ΔRST, prove that ΔXYZ, formed by joining the midpoints of the sides of ΔRST, is similar to ΔRST. • Show that if a line parallel to one side of a triangle intersects the other two sides, then the new triangle it forms is similar to the first.
G.3.C State, prove, and apply basic theorems about parallelograms. (G.2.E)	Properties may include those that address symmetry and properties of angles, diagonals, and angle sums. Students may use inductive and deductive reasoning and counter examples. Examples: • A quadrilateral is a parallelogram. • Opposite sides of a parallelogram are congruent. • Opposite angles of a parallelogram are congruent. • Diagonals of a parallelogram bisect each other. • If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. • Prove that the opposite sides of a parallelogram are congruent.
G.3.D Solve problems involving quadrilaterals and other polygons.	Examples: • On a kite, prove that one diagonal bisects the other. • Determine the measures of interior and exterior angles of a regular **polygon**. Write rules that describe the relationship between the number of sides and the angle measures.

Draft

G.3.E Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems. (G.2.H)	Example: • Determine the length of the altitude of an equilateral triangle whose side lengths measure 5 units. • If one leg of a right triangle has length 5 and the adjacent angle is 30°, what is the length of the other leg and the hypotenuse? • If one leg of a 45°-45°-90° triangle has length 5, what is the length of the hypotenuse.
G.2.F Prove and apply the Pythagorean Theorem and its converse. (G.2.F)	Examples: • Consider any right triangle with legs *a* and *b* and hypotenuse *c*. The right triangle is used to create Figures 1 and 2. Explain how these figures constitute a visual representation of a proof of the Pythagorean Theorem.  Figures 1 Figure 2 • A juice box is 6 cm by 4 cm by 8 cm. A straw is inserted into a hole in the center of the top of the box. The straw must stick out 2 cm so you can drink from it. If the straw must be long enough to touch each bottom corner of the box, what is the minimum length the straw must be? Give your answer to the nearest tenth of a centimeter.  • In ΔABC, with right angle at *C*, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem. • Apply the Pythagorean Theorem to derive the distance formula.

Draft

G.4.G Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent. (G.2.G)	Examples: • A 12-foot ladder leans against a wall to form a 63° angle with the ground. How many feet above the ground is the point on the wall at which the ladder is resting? • Use the Pythagorean Theorem to establish that $\sin^2 q + \cos^2 q = 1$ for q between 0° and 90°.

G.4.H Describe prisms, pyramids, parallelepipeds, and tetrahedra and regular polyhedra in terms of their faces, edges, vertices, and properties.

(G.2.L)

Regular polyhedra are the Platonic solids.

Descriptions could include

- characterizing basic polyhedra, such as prisms, pyramids, parallelepipeds, and tetrahedra;
- specifying the number and shape of the faces;
- specifying properties of the faces or edges;
- specifying the number of vertices and edges; or
- analyzing polyhedra by sketching a corresponding net.

G.4.I Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.

(G.2.M)

Example:

- Describe all the possible cross sections of a cube cut by a plane not parallel to a face.

Draft

G.4. Core Content: Geometry in the coordinate plane

(Geometry/Measurement, Algebra)

Students make connections between geometry and algebra by studying geometric properties and attributes that can be represented on the coordinate plane. They use the coordinate plane to represent situations that are both purely mathematical and that arise in applied contexts. **In the coordinate plane, the power of algebra can be brought to the study of geometric problems. At the same time, algebraic problems can be represented geometrically.**

Expectations	Comments/Examples
G.4.A Solve problems in coordinate geometry. (G.4.D)	Examples: • Given four points on a coordinate plane, determine whether they are the vertices of a rhombus, a square, a rectangle, a parallelogram, or none of these. • Given a parallelogram on a coordinate plane, verify that the diagonals bisect each other.
G.4.B Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line. (G.3.A)	Examples: • Write an equation for the perpendicular bisector of a given line segment. • Determine the equation of a line through the points (5, 3) and (5, –2). • Prove that the slopes of perpendicular lines are **negative inverses** of each other.
G.4.C Determine the coordinates of a point that is described geometrically. (G.3.B)	Examples: • Determine the coordinates for the midpoint of a given line segment. • Given the coordinates of three vertices of a parallelogram, determine all possible coordinates for the fourth vertex. • Given the coordinates for the vertices of a triangle, find the coordinates for the center of the circumscribed circle and the length of its radius.

G.4.D Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s). **(**G.3.C)	Examples: • Write an equation for a circle with a radius of 2 units and center at (1, 3). • Given the circle $x^2 + y^2 = 4$ and the line $y = x$, find the points of intersection. • Write an equation for a circle given a line segment as a diameter. • Write an equation for a circle determined by a given center and tangent line.

G.5. Core Content: Geometric transformations (Geometry/Measurement)

Students formalize their study of geometric transformations, focusing on the effect of such transformations and the composition of transformations on the attributes of geometric figures. They study techniques for establishing congruence and similarity by means of transformations.

May 5, 2008	Comments/Examples
G.5.A Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$. (G.4.A)	Transformations include translations, rotations, reflections, and dilations.

G.5.B Make, test, and prove (or find counterexamples to) conjectures about compositions of transformations and inverses of transformations, the commutativity and associativity of transformations, and the congruence and similarity of two-dimensional figures under various transformations. (G.4.B)	Examples: • Identify transformations (alone or in composition) that preserve congruence. • Determine whether the composition of two line reflections is commutative. • Determine whether the composition of two rotations about the same point of rotation is commutative. • Find a rotation that is equivalent to the composition of two reflections over intersecting lines. • Find the inverse of a given transformation.
G.5.C Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other. (G.4.C)	Examples: • Find a sequence of transformations that superimposes the segment with endpoints (0, 0) and (2, 1) on the segment with endpoints (4, 2) and (3, 0). • Find a sequence of transformations that superimposes the triangle with vertices (0, 0), (1, 1), and (2, 0) on the triangle with vertices (0, 1), (2, –1), and (0, –3).
G.5.D Describe the symmetries of two dimensional figures and describe transformations including reflections across a line and rotations about a point. (G.4.D)	

Draft

G.6. Core Content: *Circles and Spheres*
(Geometry/Measurement)

Students learn about the relationships among lines, angles, arcs and areas associated with circles in plane geometry. This dramatically increases the relations that can be found between items of study.

Expectation	Comments/Examples
G.6.A State, explain, and apply the basic theorems relating circles to tangents, chords, radii, secants and inscribed angles **(G.2.I)**	Examples: • If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency. • Chords equally distant from the center are congruent. • The measure of an inscribed angle is equal to half the measure of its intercepted arc. • Prove that a triangle inscribed on the diameter of a circle is a right triangle. • Prove that if a radius of a circle is perpendicular to a chord of a circle, then the radius bisects the chord.
G.6.B Derive and apply formulas for arc length and area of a sector of a circle (G.5.A)	Suggest we move this part of the standard to "additional key content" with other measurement.
G.6.C Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth. (GGM.5.F) (G.5.F)	Examples: • On a ball or globe, use a piece of string to measure the distance between two points; verify that the string lies on an arc of a great circle. • On a globe, show with examples why airlines use polar routes instead of flying due east from Seattle to Paris.

Draft

G.6.D Using straightedge and compass, construct the center of a circle from two cords, a circumscribed circle for a triangle and an inscribed circle for a triangle and explain why the constructions work. (new)	

G.7. Additional Key Content: *Measurement*

Students extend and formalize their work with geometric formulas for perimeter, area, surface area, and volume of two- and three-dimensional figures, focusing on mathematical derivations of these formulas and their applications in complex problems. They use properties of geometry and measurement to solve problems in both purely mathematical and applied contexts. Students understand the role of units in measurement and apply what they know to solve problems involving derived measures like speed or density. They understand that all measurement is approximate and specify precision in measurement problems.

March 5, 2008	Comments/Examples

Draft

G.7.A Apply formulas for surface area and volume of three-dimensional figures to solve problems. (GGM.5.B) **(G.5.B)**	Problems include those that are purely mathematical as well as those that arise in applied contexts. Three-dimensional figures include right and oblique prisms, pyramids, cylinders, cones, spheres, and composite three-dimensional figures. Example: • As Pam scooped ice cream into a cone, she began to formulate a geometry problem in her mind. If the ice cream were perfectly spherical with diameter 2.25" and sat on a geometric cone that also had diameter 2.25" and was 4.5" tall, would the cone hold all the ice cream as it melted (without her eating any of it)? She figured the melted ice cream would have the same volume as the unmelted ice cream. Find the solution to Pam's problem and justify your reasoning.
G.7.B Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures. (G.5.C)	The emphasis in high school should be on verifying the relationships between length, area, and volume and making predictions using algebraic methods. Example: • What happens to the volume of a rectangular prism if four parallel edges are doubled in length?
G.7.C Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose. (G.5.D)	Example: • The U.S. Census Bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. Saying that the U.S. population is 3 hundred million (3×10^8) is precise to one digit. Although the population had surpassed 3 hundred million by the end of that month, explain why 3×10^8 remained precise to one digit.

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G.7.D Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units. (G.5.E)	This performance expectation is intended to build on students' knowledge of proportional relationships. Students should understand the relationship between scale factors and their inverses as they relate to choices about when to multiply and when to divide in converting measurements. Derived units include those that measure speed, density, flow rates, population density, etc. Example: • A digital camera takes pictures that are 3.2 megabytes in size. If the pictures are stored on a 1-gigabyte card, how many pictures can be taken before the card is full?

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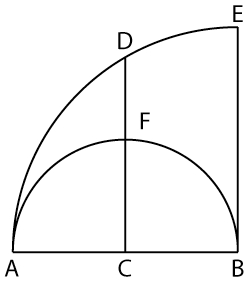
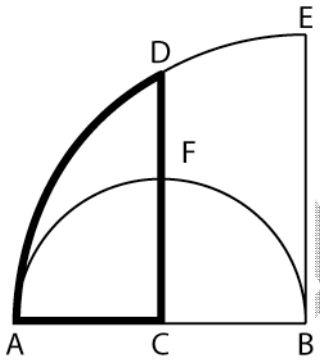
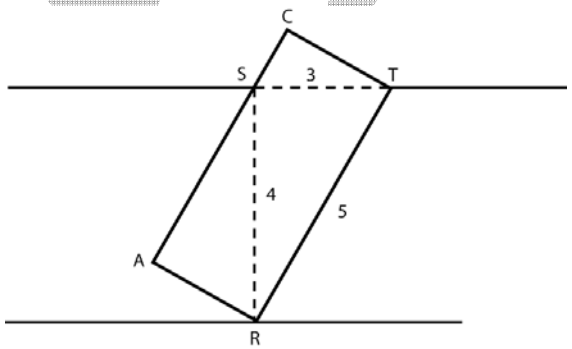
G.8. Core Processes: Solving Problems

Students formalize the development of reasoning at high school as they make and defend generalizations and justify their reasoning using accepted standards of mathematical evidence and proof. Students extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to practical and applied situations. Students use correct mathematical language, terms, and symbols as they address problems in Geometry. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an accelerating amount of information presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

May 1, 2008	Comments/Examples
G.8.A Analyze a problem situation, determine the question(s) to be answered, synthesize given information, determine how to represent the problem mathematically, and identify implicit and explicit assumptions that have been made. (PRC.1.A) **G.8.B Solve problems by formulating one or more strategies, applying the strategies, and verifying the solutions. (PRC.1.B)** **G.8.C Evaluate a solution for reasonableness and interpret the meaning of the solution in the context of the original problem. (PRC.1.C)**	Problems include those that are purely mathematical as well as those that arise in applied contexts. Descriptions of solution processes, explanations, and justifications can include numbers, words (including mathematical language), pictures, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable. Examples: • Segment AB is the diameter of the semicircle and the radius of the quarter circle shown in the figure below. Segment DC is the perpendicular bisector of segment AB. 

G.8.D **Generalize a solution strategy for a single problem to a class of related problems. (PRC.1.D)** **G.8.E** **Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics. (PRC.1.E)**	
G.8.F **Summarize and communicate mathematical ideas and reasoning with precision and efficiency in ways appropriate for the intended audience and purpose. (PRC.1.F)** **G.8.G** **Synthesize mathematical information from multiple sources to draw a conclusion, evaluate the conclusions of others, analyze a mathematical argument, and recognize flaws or gaps in reasoning. (PRC.1.G)** **G.8H** **Use inductive reasoning to make conjectures, use deductive reasoning to prove a valid conjecture, and develop a counterexample to refute an invalid conjecture. (PRC.1.H)** **Note: This Core Process is important in all of mathematics. However, in Geometry, reasoning inductively and deductively is formally developed and is evident throughout the Core Content.** **G.8.I** **Explain and demonstrate the importance of generalizations in mathematics and the role of generalizations in inductive and deductive reasoning. (PRC.1.I)**	Use inductive reasoning to make conjectures about what types of triangles are formed based upon the region where the third vertex is located. Use deductive reasoning to verify your conjectures. • Rectangular cartons that are 5 feet long need to be placed in a storeroom that is located at the end of a hallway. The walls of the hallway are parallel. The door into the hallway is 3 feet wide and the width of the hallway is 4 feet. The cartons must be carried face up. They may not be tilted. Investigate the width and carton top area that will fit through the doorway.  Generalize your results for a hallway opening of x feet and a hallway width of y feet if the maximum carton dimensions are c feet long and $x^2 + y^2 = c^2$.

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